Section 3: Chain Rule: Power Form

Composite Functions

Definition: Composite function. A function h is a composite function or is the composition of f and g if it is defined as

$$h(x) = (f \circ g)(x) = f\left[g(x)\right]$$

The domain of h is a subset of the domain of g. We must exclude any values of x for which $f\left[g(x)\right]$ is not defined.

Chain Rule

If y is a function of u given by $y = f(u)$ and u is a function of x given by $u = g(x)$, then $y(x) = f(g(x)) = (f \circ g)(x)$ has a derivative given by

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \text{ if both } \frac{dy}{du} \text{ and } \frac{du}{dx} \text{ exist.}$$

This can also be written: $y' = f'(g(x))g'(x)$.

Generalized Derivative Rules

The following rule is a special case of the general chain rule:

The derivative of a function raised to a power is $\left[(f(x))^n\right]' = n[f(x)]^{n-1} f'(x)$

Example 1: Using the Chain Rule

Find the derivative of each the following functions.

a) $y = \left(x^2 + 3x + 1\right)^3$

b) $y = \left(5x^3 + 3x^2 - 3\right)^6$

c) $y = \dfrac{1}{\left(t^2 + 2t - 3\right)^4}$

d) $y = \sqrt{3 - t}$

Example 2: Combining Rules

Find the equation of the tangent line to the graph of the function $y = x^2\sqrt{5x+1}$ at $x = 3$.

Example 3: Horizontal Tangents and the Chain Rule

Find the values of x for which the function $y = \dfrac{x^3}{(3x-2)^2}$ has horizontal tangent lines.

Example 4: Derivatives of Quotients Using the Product Rule

Find the derivative of $y = \dfrac{x^3}{(3x-2)^2}$ using the product rule.