

YOU MUST SHOW ALL WORK ON THE TEST PAPER FOR PARTIAL CREDIT!

1. Rules of Differentiation (5 points)

a) $f'(x) =$

b) If $f(x) = C$, where C is a constant, then $f'(x) =$

c) If $y = f(x) = x^n$, then $f'(x) =$

d) If $y = f(x) = ku(x)$, then $f'(x) =$

e) If $y = \ln(x)$, then $\frac{dy}{dx} =$

f) If $y = \log_b(x)$, then $\frac{dy}{dx} =$

g) If $y = b^x$, then $\frac{dy}{dx} =$

h) If $y = f(x) = u(x) \pm v(x)$, then $f'(x) =$

i) If $y = f(x) = F(x)S(x)$, then $f'(x) =$

j) If $y = f(x) = \frac{T(x)}{B(x)}$, then $f'(x) =$

k) If $u(x)$ is a differentiable function of x , n is any real number, and

$$y = f(x) = [u(x)]^n, \text{ then } f'(x) =$$

l) If $u(x)$ is a differentiable function of x , n is any real number, and

$$y = \ln[u(x)], \text{ then } \frac{dy}{dx} =$$

m) If $u(x)$ is a differentiable function of x , n is any real number, and

$$y = e^{u(x)}, \text{ then } \frac{dy}{dx} =$$

Cost Analysis (6 points)

2. The total cost (in dollars) of producing x television sets is

$$C(x) = 9,875 + 205x - 0.1x^2.$$

- a) Find the *exact* cost of producing the 78th television set.
- b) Use the marginal cost to **approximate the cost** of producing the 78th television set.

3. **The Derivative (8 points)** Find the derivative of each function but do not simplify.

a) $g(x) = (2x^4 + 3x^2 - 5)(x^3 + x + 5)$

b) $g(x) = \frac{3x^2 - 5}{x^2 + x + 5}$

3. **The Derivative (8 points)** Find the indicated derivative of each of the following functions

a) y' if $y = 2(x^2 - 3x)^5$

b) $\frac{dg}{dt}$ if $g(t) = \sqrt[3]{3t^4 + 1}$

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4. **Compound Interest** Find the amount in 25 years if \$11,000 is placed in an account earning 7.25%

a) Compounded weekly

b) Compounded continuously

5. Rules of Differentiation (12 points)

Find the derivative of the following functions (Do not simplify):

a) $f'(x)$ for $f(x) = \frac{3}{(1-4x)^2}$.

b) $f'(x)$ for $f(x) = (2x^3 - x^2)^2(x^2 + 2x^3)$

c) $y = e^{3x^2 - 11x + 5}$

d) $y = \ln(x^3 + 3x^2 - 2)$

6. **Marginal Analysis (14 points)** The market research department of a furniture manufacturing company recommends that the company manufacture and market, a new model of chair. After suitable test marketing, the marketing department presents the following demand equation:

$$x = 62,400 - 120p,$$

where x is the demand at $\$p$ per chair.

- a) Solve the demand equation for p in terms of x .
- b) The financial department provides the following cost function:

$$C(x) = 6,000 + 50x$$

Find the marginal cost.

- c) Find the revenue equation in terms of x .
- d) Find the marginal revenue.
- e) Find $R'(10,000)$ and interpret the results.
- f) Find the profit equation in terms of x .
- g) Find the marginal profit function.

(9 points)

8. **Derivatives of Logarithmic and Exponential Functions with Base e** Find the derivative of the following functions:

a) $f(x) = (x^2 + 1)\ln(2x + 1)$

b) $f(x) = \frac{x^2 - 1}{\ln x}$

c) $f(x) = \frac{e^x - 1}{e^x + 1}$

8. **Partial Derivatives (6 points)** Find the indicated partial derivatives of the function $z = f(x, y) = 5x^3y + xy^3 - 2y^4$

a) f_x

b) $\frac{\partial f}{\partial x}$

(6 points)

9. **Derivatives of General Log and Exponential Functions** Find $f'(x)$ for the following functions:

a) $f(x) = \log_4 x$

b) $f(x) = 4^x$

(6 points)

10. **Implicit Differentiation** The automobile assembly plant you manage has the Cobb-Douglas production function

$$P = x^{0.3}y^{0.7},$$

where P is the number of automobiles it produces per year, x is the number of employees and y is the daily operating budget (in dollars). Assume a production level of 1000 automobiles per year.

a) Find $\frac{dy}{dx}$.

b) Evaluate at $x = 80$ and interpret the answer.

(6 points)

12. **Chain Rule** Empire Electronics has an annual profit given by the profit equation $P = -200,000 + 4,000x - 0.46x^2 - 0.00001x^3$, where x is the number of computers it sells per year. The number of computers it can manufacture per year depends on the number n of assembly line workers employed, according to the formula: $x = 100n$. Use the chain rule to find $\left. \frac{dp}{dn} \right|_{n=25}$, and interpret the result.

(6 points)

13. **Related Rates** Suppose that for a company manufacturing calculators, the cost and revenue equations are given by $C = 62,000 + 50x$ and $R = 300x - \frac{x^2}{40}$, respectively. Suppose the production output in one week is x calculators. If production is increasing at a rate of 100 calculators per week at a production rate of 1,000 calculators per week, find the rate of increase (decrease) in:

a) cost

b) revenue

c) profit