

1. Rules of Differentiation (4 points)

a) $f'(x) =$

b) If $f(x) = C$, then $f'(x) =$

c) If $y = f(x) = x^n$, then $f'(x) =$

d) If $y = f(x) = ku(x)$, then $f'(x) =$

e) If $y = f(x) = u(x) \pm v(x)$, then $f'(x) =$

f) If $y = f(x) = F(x)S(x)$, then $f'(x) =$

g) If $y = f(x) = \frac{T(x)}{B(x)}$, then $f'(x) =$

h) If $u(x)$ is a differentiable function of x , n is any real number, and

$$y = f(x) = [u(x)]^n, \text{ then } f'(x) =$$

Cost Analysis (6 points)2. The total cost (in dollars) of producing x television sets is

$$C(x) = 10,100 + 195x - 0.1x^2.$$

a) Find the *exact* cost of producing the 51st television set.b) Use the marginal cost to approximate the cost of producing the 51st television set.

Revenue, Cost, Profit (15 points)

3. The market research department of a guitar manufacturing company recommends that the company manufacture and market, a new model of electric guitar. After suitable test marketing, the marketing department presents the following demand equation:

$$x = 36,000 - 40p ,$$

where x is the demand at $\$p$ per guitar.

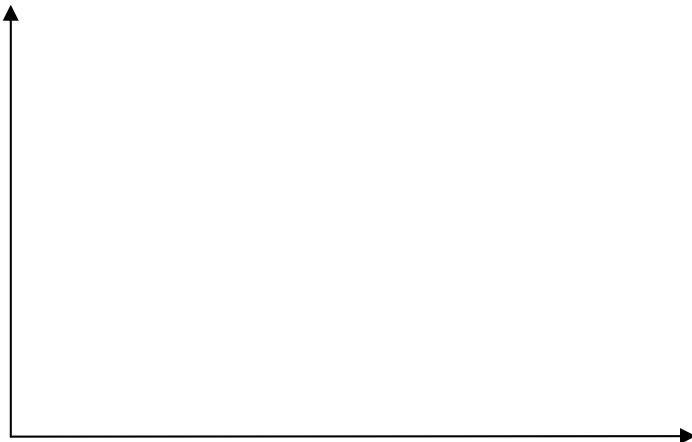
- a) Solve the demand equation for p .
- b) The financial department provides the following cost function:

$$C(x) = 8,400 + 275x$$

Find the marginal cost.

- c) Find the revenue equation in terms of x .
- d) Find the marginal revenue.
- e) Find $R'(2,000)$ and $R'(3,000)$, and interpret the results.

- f) Find the profit equation in terms of x .
- g) Find the marginal profit function.
- h) Find the production level that will produce a maximum profit.
- i) Find the maximum profit.
- j) Find the price of the new electric guitar that will produce the maximum profit.
- k) Sketch the revenue and cost functions below and label areas of profit, areas of loss, and the break-even points.



Derivatives (6 points)

4. Find the indicated derivative of each function:

a) y' if $y = 2\sqrt{x^3 + 3x} - 2x^{-5}$

b) $\frac{ds}{dt}$ if $s(t) = 2(4 - 5t^3)^4$

The Second Derivative (6 points)

5. Find the second derivative of each function:

a) $f(x) = 3x^2 - 5\sqrt{x} - 2x^{\frac{1}{2}}$

b) $g(x) = \frac{4x - 2}{5x + 3}$

Marginal Analysis (10 points)

4. The total cost (in dollars) of manufacturing x umbrellas is given by the function

$$C(x) = 9,000 + 2x, \quad 0 \leq x \leq 2,500.$$

- a) Find the marginal cost function.
- b) Find the average cost function $\overline{C}(x)$.
- c) Find the marginal average cost function.
- d) Evaluate the marginal average cost function for a production level of 1,000 umbrellas.
- e) Interpret the result in d).
- f) Does the average cost function you found in (b) have a horizontal asymptote? If it does, interpret what this means. If the function doesn't have a horizontal asymptote, explain why you think it doesn't.

5. Rules of Differentiation. (8 points)

Find the derivative of the following functions (Do not simplify):

a) $f'(x)$ for $f(x) = \frac{(x^2 + 3)^3}{3 - 7x}$.

b) $f'(x)$ for $f(x) = (x^2 - 5)^5(2x^3 - 5)^3$

Asymptotes (6 points)

6. Find the vertical and horizontal asymptotes (if they exist) for the graph of

$$f(x) = \frac{3x^2 + 5x - 2}{x^2 - 4}$$

The Derivative and Graphs (10 points)

7. Let $f(x) = 8x^3 - 2x^4$.

a) Find the domain of f .

b) Find $f'(x)$.

c) For what intervals is the graph of f increasing? Decreasing?

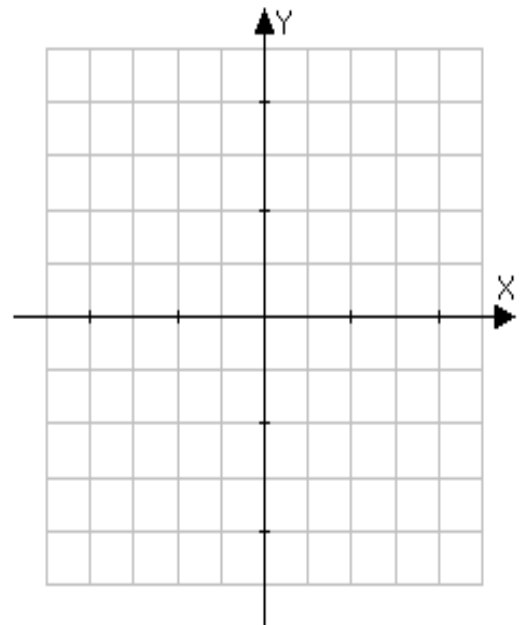
d) Find any local maxima and local minima.

e) Find $f''(x)$.

f) For what intervals is the graph of f concave upward? Concave downward?

g) Find any inflections points on the graph of f .

a) Sketch the graph of the function f .



Curve Sketching Techniques (10 points)

8. Let $f(x) = \frac{x}{(x+2)^2}$.

a) Find $f'(x)$.

b) Find $f''(x)$.

c) Find the domain of f .

d) Sketch the graph of the function f on the axes below.

b) For what intervals is the graph f increasing? Decreasing?

c) For what intervals is the graph of f concave upward? Concave downward? Is there an inflection point? If so, where?

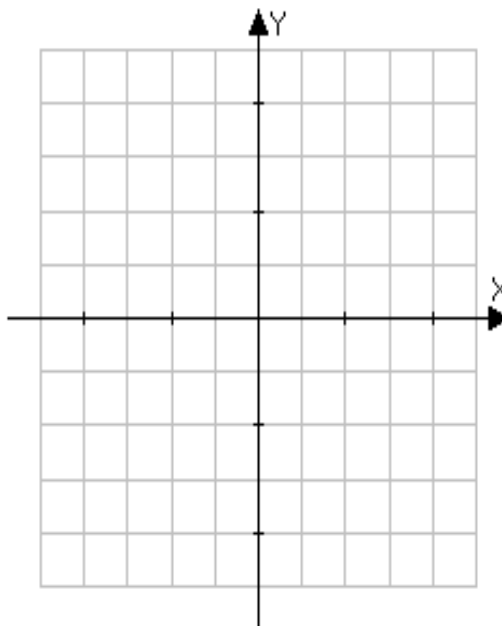
d) Find the x -intercept.

e) Find the y -intercept.

f) Find the horizontal asymptote.

g) The vertical asymptote(s) is/are _____

h) Sketch the graph of the function f .



Absolute Values (6 points)

9. Find the absolute minimum value of the function $f(x) = 2x + \frac{5}{x} + \frac{4}{x^3}$ on the interval $(0, \infty)$.

Partial Derivatives (6 points)

10. Find the indicated partial derivatives of the function

$$z = f(x, y) = 7x^2y^3 - 2x^3 + 3y$$

a) $\frac{\partial z}{\partial x}$

b) $f_y(x, y)$

c) $f_{xy}(x, y)$

(7 points)

- 11. Optimization** A car rental agency rents 300 cars per day at a rate of \$29 per day. For each \$1.50 increase in rate, 6 fewer cars are rented. At what rate should the cars be rented to produce the maximum income? What is the maximum income?

Name _____

MATH 1425
Business Calculus 1
Test 2
Mike Huff
Fall 2010

Tools: Graphing Calculator

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