**YOU MUST SHOW ALL WORK _ON THE TEST PAPER_ FOR PARTIAL CREDIT!**

**1. Rules of Differentiation (5 points)**

a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

b) If $f(x) = C$, where $C$ is a constant, then $f'(x) = 0$

c) If $y = f(x) = x^n$, then $f'(x) = nx^{n-1}$

d) If $y = f(x) = ku(x)$, then $f'(x) = ku'(x)$

e) If $y = \ln(x)$, then $\frac{dy}{dx} = \frac{1}{x}$

f) If $y = \log_b(x)$, then $\frac{dy}{dx} = \frac{1}{x \ln b}$

g) If $y = b^x$, then $\frac{dy}{dx} = \ln b \cdot b^x$

h) If $y = f(x) = u(x) \pm v(x)$, then $f'(x) = u'(x) \pm v'(x)$

i) If $y = f(x) = F(x)S(x)$, then $f'(x) = F(x)S'(x) + S(x)F'(x)$.

j) If $y = f(x) = \frac{T(x)}{B(x)}$, then $f'(x) = \frac{B(x)T'(x) - T(x)B'(x)}{[B(x)]^2}$

k) If $u(x)$ is a differentiable function of $x$, $n$ is any real number, and

$y = f(x) = [u(x)]^n$, then $f'(x) = n[u(x)]^{n-1} u'(x)$

l) If $u(x)$ is a differentiable function of $x$, $n$ is any real number, and

$y = \ln[u(x)]$, then $\frac{dy}{dx} = \frac{1}{u(x)} u'(x)$

m) If $u(x)$ is a differentiable function of $x$, $n$ is any real number, and

$y = e^{u(x)}$, then $\frac{dy}{dx} = e^{u(x)} u'(x)$

**Cost Analysis (6 points)**

**2.** The total cost (in dollars) of producing $x$ television sets is

$$C(x) = 9,875 + 205x - 0.1x^2.$$

a) Find the ***exact*** **cost** of producing the 78[th] television set.

$$C_{78} = C(78) - C(77)$$
$$= 25256.6 - 25067.1 = \$189.50$$

b) Use the marginal cost to **approximate the cost** of producing the 78[th] television set.

$$C'(x) = 205 - 0.2x$$

$$C_{78} \approx C'(77) = 205 - 0.2(77) = \$189.60$$

**3. The Derivative (8 points)** Find the derivative of each function but do not simplify.

a) $g(x) = (2x^4 + 3x^2 - 5)(x^3 + x + 5)$

$$g'(x) = (2x^4+3x^2-5)(3x^2+1) + (x^3+x+5)(8x^3+6x)$$

b) $g(x) = \dfrac{3x^2 - 5}{x^2 + x + 5}$

$$g'(x) = \frac{(x^2+x+5)(6x) - (3x^2-5)(2x+1)}{(x^2+x+5)^2}$$

**3. The Derivative (8 points)** Find the indicated derivative of each of the following functions

a) $y'$ if $y = 2(x^2 - 3x)^5$

$$y' = 2\left[5(x^2-3x)^4(x^2-3x)'\right]$$
$$= 10(x^2-3x)^4(2x-3)$$

b) $\frac{dg}{dt}$ if $g(t) = \sqrt[3]{3t^4+1}$ $= (3t^4+1)^{1/3}$ $\qquad (u^{1/3})' = \frac{1}{3}u^{-2/3}u'$

$$g'(t) = \frac{1}{3}(3t^4+1)^{-2/3}(3t^4+1)'$$
$$= \frac{12t^3}{3(3t^4+1)^{2/3}} = \frac{4t^3}{(3t^4+1)^{2/3}}$$

(8

**4. Compound Interest** Find the amount in 25 years if $11,000 is placed in an account earning 7.25%

a) Compounded weekly $n=52$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= 11000\left(1+\frac{.0725}{52}\right)^{52\cdot 25}$$
$$= \$67298.16$$

b) Compounded continuously

$$A = Pe^{rt}$$
$$= 11000e^{.0725(25)}$$
$$= \$67383.17$$

**5. Rules of Differentiation (12 points)**

Find the derivative of the following functions (Do not simplify):

a) $f'(x)$ for $f(x) = \dfrac{3}{(1-4x)^2} = 3(1-4x)^{-2}$

$$f'(x) = 3\left[-2(1-4x)^{-3}(1-4x)'\right]$$

$$= -6(1-4x)^{-3}(-4)$$

$$= \frac{24}{(1-4x)^3}$$

use the chain rule!

b) $f'(x)$ for $f(x) = (2x^3 - x^2)^2 (x^2 + 2x^3)$

$$f'(x) = (2x^3-x^2)^2(2x+6x^2) + (x^2+2x^3)\left[2(2x^3-x^2)(6x^2-2x)\right]$$

c) $y = e^{3x^2-11x+5}$ $\qquad y' = (6x-11)e^{3x^2-11x+5}$

d) $y = \ln(x^3 + 3x^2 - 2)$ $\qquad y' = \dfrac{3x^2+6x}{x^3+3x^2-2}$

$y = \ln u$

$\dfrac{dy}{dx} = \dfrac{1}{u}u'$

6. **Marginal Analysis (14 points)** The market research department of a furniture manufacturing company recommends that the company manufacture and market, a new model of chair. After suitable test marketing, the marketing department presents the following demand equation:

$$x = 62,400 - 120p,$$

$120p = 62400 - x$

$p = 520 - \frac{1}{120}x$

where $x$ is the demand at $\$p$ per chair.

a) Solve the demand equation for $p$ in terms of $x$.

$p = 520 - \frac{1}{120}x$

b) The financial department provides the following cost function:

$$C(x) = 6,000 + 50x$$

Find the marginal cost.

$C'(x) = 50$

c) Find the revenue equation in terms of $x$. $R(x) = xp$

$= 520x - \frac{1}{120}x^2$

d) Find the marginal revenue.

$R'(x) = 520 - \frac{1}{60}x$

e) Find $R'(10,000)$ and interpret the results.

$R'(10,000) = 520 - \frac{1}{60}10,000 = 353.\overline{33}$

when 10,000 units are made the next one results in about $353.33 of additional revenue.

f) Find the profit equation in terms of $x$.

$P(x) = 520x - \frac{1}{120}x^2 - 6000 - 50x$

g) Find the marginal profit function.

$= -\frac{1}{120}x^2 + 470x - 6000$

$P'(x) = -\frac{1}{60}x + 470$

(9 points)

**8. Derivatives of Logarithmic and Exponential Functions with Base e** Find the derivative of the following functions:

product

a) $f(x) = (x^2 + 1)\ln(2x + 1)$

$$f'(x) = (x^2+1)\frac{2}{2x+1} + \ln(2x+1)(2x)$$

quotient

b) $f(x) = \dfrac{x^2 - 1}{\ln x}$

$$f'(x) = \frac{(\ln x)(2x) - (x^2-1)\frac{1}{x}}{(\ln x)^2}$$

quotient

c) $f(x) = \dfrac{e^x - 1}{e^x + 1}$

$$f'(x) = \frac{(e^x+1)(e^x) - (e^x-1)e^x}{(e^x+1)^2}$$

**8. Partial Derivatives (6 points)** Find the indicated partial derivatives of the function $z = f(x, y) = 5x^3y + xy^3 - 2y^4$

a) $f_x$

$$f_x = 15x^2y + y^3$$

b) $\dfrac{\partial f}{\partial \cancel{x} y}$

Should have been a y!

$$f_y = 5x^3 + 3xy^2 - 8y^3$$

(6 points)

9. **Derivatives of General Log and Exponential Functions** Find $f'(x)$ for the following functions:

a) $f(x) = \log_4 x$

$$f'(x) = \frac{1}{x \ln 4}$$

b) $f(x) = 4^x$

$$f'(x) = \ln x \cdot 4^x$$

(6 points)

10. **Implicit Differentiation** The automobile assembly plant you manage has the Cobb-Douglas production function

$$P = x^{0.3}y^{0.7},$$

where P is the number of automobiles it produces per year, *x* is the number of employees and *y* is the daily operating budget (in dollars). Assume a production level of 1000 automobiles per year.

a) Find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left(1000 = x^{0.3}y^{0.7}\right) \leftarrow \text{product rule}$$

$$0 = x^{0.3}\,.0.7\,y^{-0.3}y' + 0.3\,x^{-0.7}y^{0.7}$$

Solve for $y'$!

$$y' = -\frac{0.3\,x^{-0.7}y^{0.7}}{0.7\,x^{0.3}y^{-0.3}}$$

$$y' = -3\frac{y}{x}$$

b) Evaluate at $x = 80$ and interpret the answer.

we need y also

$$1000 = 80^{0.3}y^{0.7}$$

$$y = \left(\frac{1000}{80^{0.3}}\right)^{1/0.7}$$

$$\approx 2951.918$$

$$y' = -3\,\frac{2951.918}{80}$$

$$= -110.70$$



Add ing the 81st employee will lower operating expenses ~ $110.70.

**(7 points)**

**11. Elasticity of Demand** Given the demand equation $p + \frac{3}{100}x = 12$, $0 \le p \le 12$, answer the following:

$100p + 3x = 1200$

$3x = 1200 - 100p$

$x = 400 - \frac{100}{3}p$.

a) Express the demand $x$ as a function of the price $p$.

$f(p) = x = 400 - \frac{100}{3}p$

b) Express the revenue $R$ as a function of the price $p$.

$R(p) = xp = \left(400 - \frac{100}{3}p\right)p = 400p - \frac{100}{3}p^2$

c) Find the point elasticity of demand, $E(p)$.

$$E(p) = \frac{-pf'(p)}{f(p)} = \frac{\left[-p\left(-\frac{100}{3}\right)\right]3}{\left[400 - \frac{100}{3}p\right]3} = \frac{100p}{1200-100p} = \frac{p}{12-p}$$

d) For which values of $p$ is the demand inelastic? Elastic?

inelastic if $0 \le p < 6$

elastic if $6 < p \le 12$

e) If $p = \$4$, what is the approximate change for a 10% price increase? Will revenue increase or decrease?

$E(4) = \frac{4}{12-4} = \frac{4}{8} = \frac{1}{2}$

$\frac{1}{2}(10\%) = 5\%$ So a 10% increase in price causes a 5% decrease in demand and revenue goes up.

(6 points)

12. **Chain Rule** Empire Electronics has an annual profit given by the profit equation $P = -200{,}000 + 4{,}000x - 0.46x^2 - 0.00001x^3$, where $x$ is the number of computers it sells per year. The number of computers it can manufacture per year depends on the number $n$ of assembly line workers employed, according to the formula: $x = 100n$ Use the chain rule to find $\left.\frac{dp}{dn}\right|_{n=25}$, and interpret the result.

$n = 25$

$x = 100n$

$x = 100(25) = 2500$

$P = f(x)$

$x = g(n)$

$\frac{dP}{dx} = P' = 4000 - 0.92x - .00003x^2$

$P'(2500) = 1512.5$

$\frac{dx}{dn} = 100$

$\frac{dP}{dn} = \frac{dP}{dx} \cdot \frac{dx}{dn}$

$= 1512.5(100)$

$= 151{,}250$

Hiring the 26th worker would increase profit by about $151,250.

(6 points)

13. **Related Rates** Suppose that for a company manufacturing calculators, the cost and revenue equations are given by $C = 62{,}000 + 50x$ and $R = 300x - \frac{x^2}{40}$, respectively. Suppose the production output in one week is $x$ calculators. If production is increasing at a rate of 100 calculators per week at a production rate of 1,000 calculators per week, find the rate of increase (decrease) in:

time rate of change

$\frac{dx}{dt} = 100$

$x = 1000$

a) cost

$\frac{dC}{dt}$

$\frac{dC}{dt} = 50 \cdot \frac{dx}{dt} = 50(100) = 5000/\text{wk}$

b) revenue

$\frac{dR}{dt}$

$\frac{dR}{dt} = 300\frac{dx}{dt} - \overbrace{\frac{1}{20} \cdot x \cdot \frac{dx}{dt}}^{\text{I used the chain rule}}$

$= 300(100) - \frac{1}{20}(1000)100 = \$25{,}000/\text{wk}$

c) profit

$\frac{dP}{dt}$

$P = R - C \Rightarrow$

$\frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt} = 25{,}000 - 5{,}000 = \$20{,}000/\text{wk}$