Name_____ BUSINESS CALCULUS I

1. **The Derivative** Find the indicated derivative of each of the following functions

a) y' if $y = 7(x^3 - 3)^5$ $y' = 35(x^3-3)^4(3x^2)$

b) $\frac{dg}{dw}$ if $g(w) = \sqrt[3]{w^2+1}$ $= (w^2+1)^{1/3}$ $\frac{dg}{dw} = \frac{1}{3}(w^2+1)^{-2/3}(2w)$

2. **Compound Interest** If \$1,000 is invested at an annual interest rate r compounded monthly, the amount in the account at the end of 4 years is given by

$$A(r) = 1{,}000\left(1 + \frac{1}{12}r\right)^{48}$$

Find the rate of change of the amount A with respect to the interest rate r. $A'(r) = 48000\left(1+\frac{1}{12}r\right)^{47}\left(\frac{1}{12}\right)$

3. **The Second Derivative**

Find the second derivative of each function:

a) $f(x) = 4x^3 - 2\sqrt{x} - 2x^{-3}$ $f'(x) = 12x^2 - x^{-1/2} + 6x^{-4}$ $f''(x) = 24x + \frac{1}{2}x^{-3/2} - 24x^{-5}$

b) $g(x) = \frac{5x-3}{3x+5}$

$g'(x) = \frac{(3x+5)5 - (5x-3)3}{(3x+5)^2}$

$= \frac{34}{(3x+5)^2}$

$= 34(3x+5)^{-2}$

$g''(x) = -68(3x+5)^{-3}(3)$

4. Marginal Analysis

The market research department of a guitar manufacturing company recommends that the company manufacture and market, a new model of acoustic guitar. After suitable test marketing, the marketing department presents the following demand equation:

$$x = 27{,}000 - 90p\,,$$

where x is the demand at $\$p$ per guitar.

a) Solve the demand equation for p. $p = 300 - \frac{1}{90}x$

b) The financial department provides the following cost function:

$$C(x) = 10{,}400 + 75x \qquad C'(x) = 75$$

Find the marginal cost.

c) Find the revenue equation in terms of x. $R(x) = 300x - \frac{1}{90}x^2$

d) Find the marginal revenue. $R'(x) = 300 - \frac{1}{45}x$

e) Find $R'(15{,}000)$ and $R'(10{,}000)$, and interpret the results.

f) Find the profit equation in terms of x.

g) Find the marginal profit function.

h) Find the production level that will produce a maximum profit.

i) Find the maximum profit.

j) Find the price of the new acoustic guitar that will produce the maximum profit.

5. Rules of Differentiation

Find the derivative of the following functions (Do not simplify):

a) $f'(x)$ for $f(x) = \dfrac{x^2}{(3-7x)^3}$.

b) $f'(x)$ for $f(x) = (x^3 - 5x^{\frac{1}{3}})^5$

c) $f'(x)$ for $f(x) = \left[5x^4 - 3x\right]^3\left(7x^3 + 7x^2\right)$.

6. Marginal Analysis

The total cost (in dollars) of manufacturing x acoustic guitars is given by the function

$$C(x) = 11{,}500 + 65x\,,\ 1 \le x \le 10{,}000\,.$$

a) Find the average cost per guitar if 8000 guitars are produced.

b) Find the average cost function $\overline{C}(x)$. $\overline{C}(x) = 65 + \frac{11500}{x}$

c) Find the marginal average cost function. $\overline{C}'(x) = -\frac{11500}{x^2}$

d) Evaluate the marginal average cost function for a production level of 5,000 guitars. -0.00046

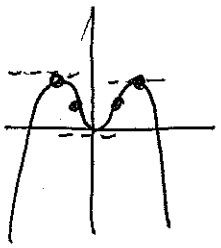
e) Interpret the result in d). cost goes down

f) Does the average cost function you found in (b) have a horizontal asymptote? If it does, interpret what this means. If the function doesn't have a horizontal asymptote, explain why you think it doesn't.

yes, $y = 65$ average cost goes to \$65 which is the variable cost per guitar.

7. The Second Derivative and Graphs

Let $f(x) = 2x^2 - x^4$.



a) Find any local maxima, local minima, and inflections points on the graph of f. max @ $x = \pm 1$ min @ $0 = x$

b) Sketch the graph of the function f, include the tangent line at each local maxima or minima.

c) For what intervals is the graph of f increasing? Decreasing? inc: $(-\infty,-1) \cup (0,1)$ dec: $(-1,0) \cup (1,\infty)$

d) For what intervals is the graph of f concave upward? Concave downward? cu: $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ cd: $\left(-\infty, -\frac{1}{\sqrt{3}}\right) \cup \left(\frac{1}{\sqrt{3}},\right.$

8. Find the **vertical and horizontal asymptotes** (if they exist) for the graph of $f(x) = \dfrac{x^2 + 5x + 6}{x^2 - 9}$ VA: $x = 3$ {Hole: @ $x = -3$} HA: $y = 1$

9. Partial Derivatives Find the indicated partial derivatives of the function

$z = f(x,y) = 6x + 3y^4 - 2x^2y$

a) $\dfrac{\partial z}{\partial x} = 6 - 4xy$

b) $f_{xx}(x,y) = -4y$

c) $f_{xy}(x,y) = -4x$