Section 2: The Derivative

Definition: The **derivative** f' **of the function** f **at** c is given by

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

if the limit exists. We can also write the derivative with the notation

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Example 1: Finding the Derivative of a Polynomial

Find the derivative of $f(x) = x^2 - 3x + 4$ at $x = c$.

Example 2: Finding the Derivative of a Root Function

Find the derivative of $f(x) = \sqrt{x}$ at $x = 2$ using both formulas.

Facts:

1. The tangent line to $y = f(x)$ at $x = c$ goes through $(c, f(c))$ and has slope $f'(c)$.
2. The derivative $f'(c)$ is the instantaneous rate of change of y with respect to x. That is the derivative $f'(c)$ is the velocity at $x = c$ if $y = f(x)$ is a position function and speed is $\left|f'(c)\right|$.

Example 3: Finding the Equation of a Tangent Line

Find an equation of the tangent line to $f(x) = x^2 - 3x + 4$ at $(2,2)$.

Example 4: Interpreting the Derivative as Instantaneous Velocity

Given the position function $f(t) = 2t^3 - t + 1$, find the velocity $f'(c)$ and the speed $\left|f'(c)\right|$.