The Relationship between the Graphs of f , f' , and f''

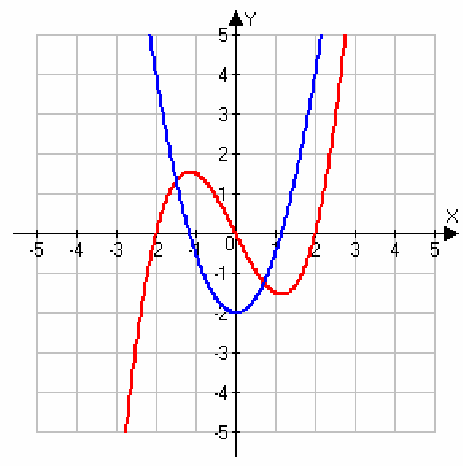
Increasing and Decreasing Functions

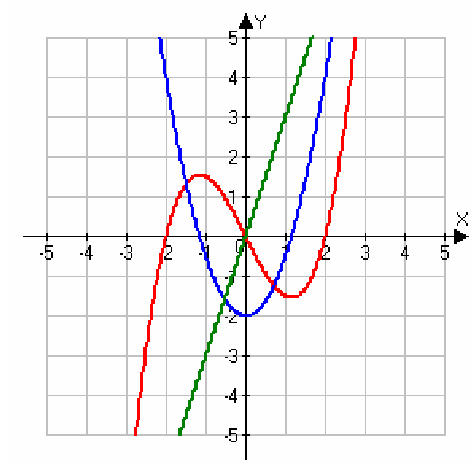
Definition: A function f is **increasing on an interval** (a,b) if $f(x_2) > f(x_1)$ whenever $a < x_1 < x_2 < b$; f is **decreasing on an interval** (a,b) if $f(x_2) < f(x_1)$ whenever $a < x_1 < x_2 < b$.

Theorem: a) If $f'(x) > 0$ on an interval (a,b), then f is increasing on (a,b).

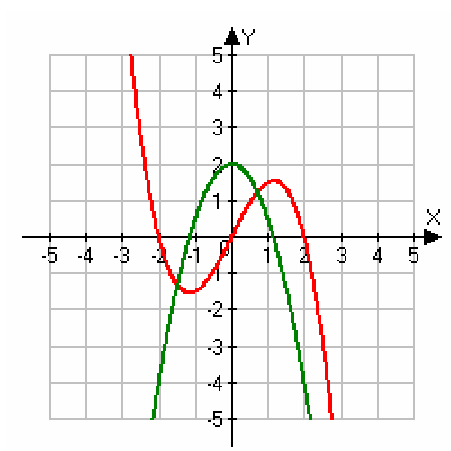
b) If $f'(x) < 0$ on an interval (a,b), then f is decreasing on (a,b).

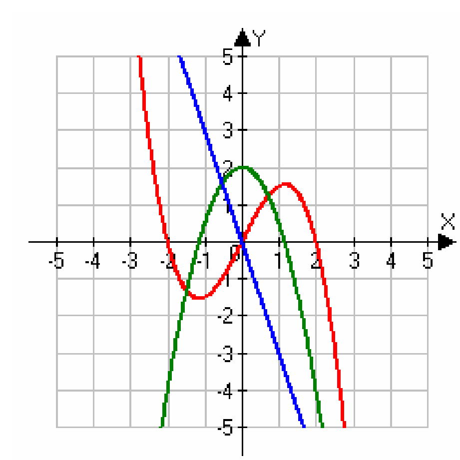
Example 1: The Derivative and Graphs





Example 2: The Derivative and Graphs





Definition:

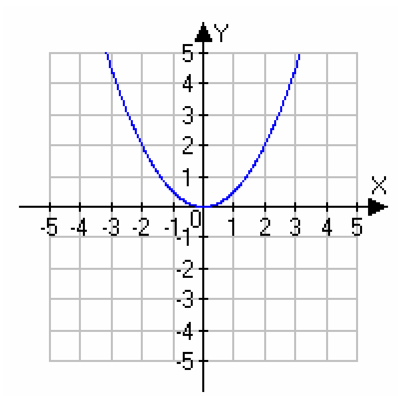
a) If $f(c) \le f(x)$ for all x in some interval (a,b), then $f(c)$ is a **local** (or relative) **minimum** value for the function f.

b) If $f(c) \ge f(x)$ on an interval (a,b), then $f(c)$ is a **local** (or relative) **maximum** value for the function f.

c) The **critical values** (or **critical numbers**) of f are the values of x in the domain of f where $f'(x) = 0$ or $f'(x)$ does not exist.

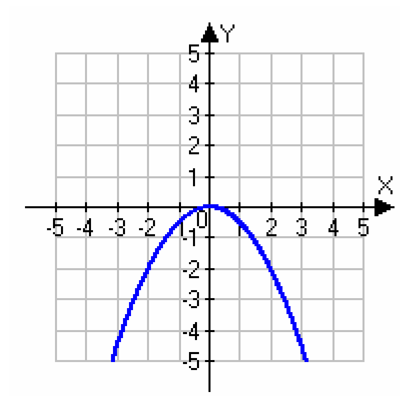
Concavity

Definition: If $f'(x)$ is increasing on an interval (a,b), then the function f is said to be **concave upward** on (a,b). : If $f'(x)$ is decreasing on an interval (a,b), then the function f is said to be **concave downward** on (a,b).



Concave upward

f' is increasing



concave downward

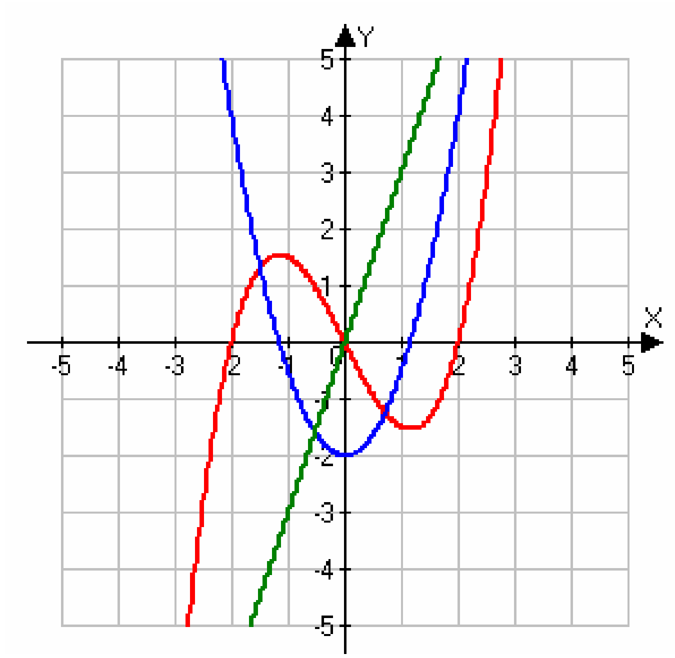
f' is decreasing

How do we determine if f' is increasing or decreasing? By looking at the derivative of the derivative, the second derivative, of course.

Test for Concavity: If for each $x \in (a,b)$, we have:

a) $f''(x) > 0$, then $f'(x)$ is increasing on (a,b) and the graph of f is concave upward (smiling or bending to the left).

b) $f''(x) < 0$, then $f'(x)$ is decreasing on (a,b) and the graph of f is concave downward (frowning or bending to the right).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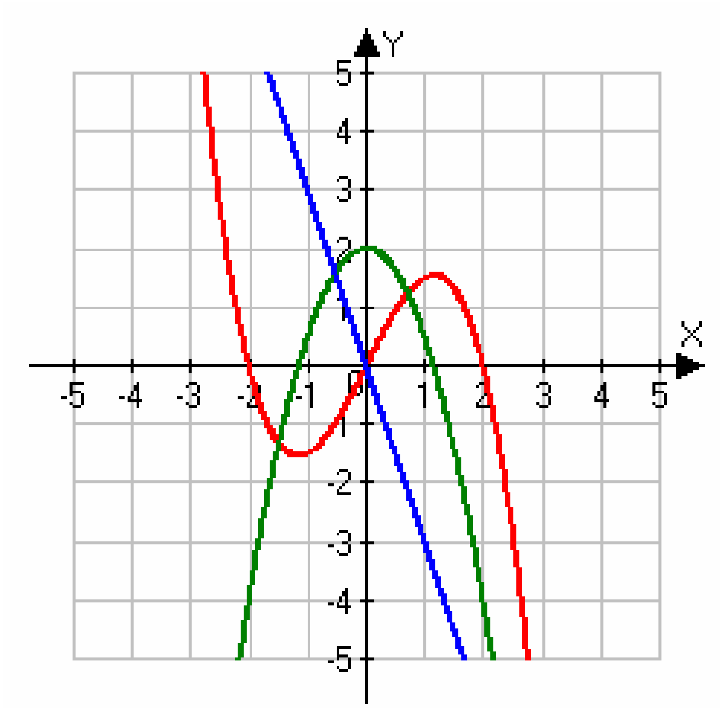
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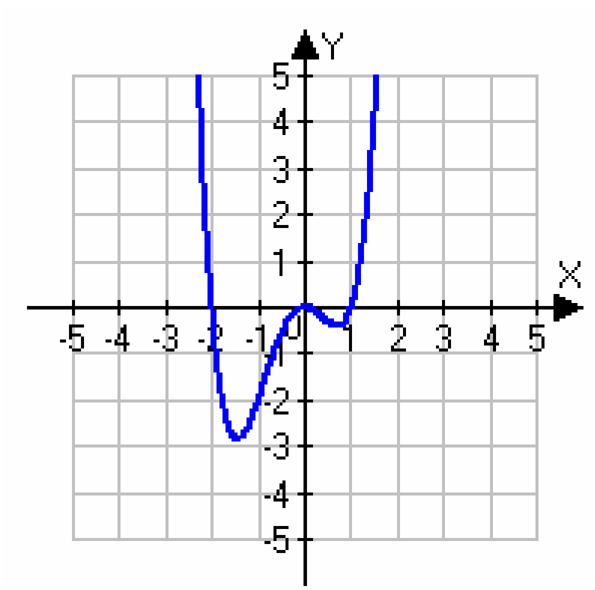
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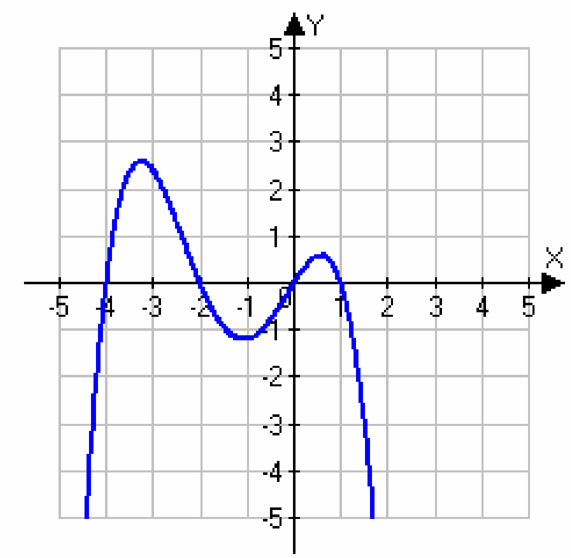
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-1
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Name ____ Date ____________

Calculus 1
Group Work
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