

1. Suppose the height of a projectile fired vertically upward from a height of 1024 feet with an initial velocity of 256 feet per second is given by

$$h(t) = -16t^2 + 256t + 1024.$$

What is the instantaneous velocity of the object after 8 seconds?

$$h'(t) = -32(t) + 256$$

$$h'(8) = -32(8) + 256$$

$$h'(8) = 0 \text{ ft/sec}$$

2. A company that manufactures and sells paint estimates that it can sell  $g(p)$  gallons of paint at a price of  $p$  dollars per gallon. Interpret the following expressions in terms of gallons and dollars.

a)  $g(10) = 15,500$

**At \$10.00 per gallon, the company expects to sell 15,500 gallons of paint**

b)  $g^{-1}(16000) = 9.50$

**The company sells 16,000 gallons of paint when the price is \$9.50 per gallon**

c)  $g'(8.5) = 0$

**At \$8.50 per gallon, there is no change in the quantity of paint sold. The quantity sold is either a maximum or a minimum.**

- d) If  $g'(11) = -200$ , what can you say about the effect of increasing the price of paint from \$11 per gallon to \$12 per gallon?

**When going from \$11.00 to \$12.00 per gallon the company sells 200 gallons less paint.**

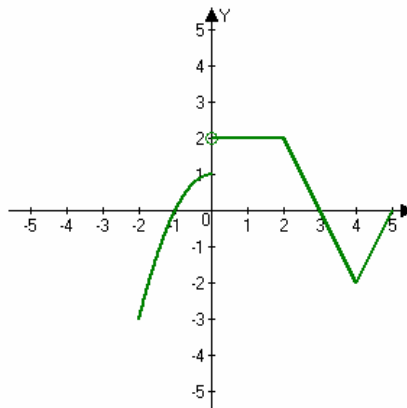
e) What are the units of  $g'(p) = \frac{dg}{dp}$ ? Gallons per dollar

3. Use the graph of the function  $f(x)$  below to answer the following questions:

- a) For what values of  $x$  is the function not differentiable? **0, 2, 4 (-2, 5 ?)**

b) What is  $f'(1)$ ? **0**

c) What is  $f'(3)$ ? **-2**



d) Make a rough sketch the derivative of  $f$  below.

4. Use the definition of the derivative to find  $f'(3)$  where  $f(x) = -x^2 + 3x + 2$ .

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{-(x+h)^2 + 3(x+h) + 2 - (-x^2 + 3x + 2)}{h} \\&= \lim_{h \rightarrow 0} \frac{-x^2 - 2xh - h^2 + 3x + 3h + 2 + x^2 - 3x - 2}{h} \\&= \lim_{h \rightarrow 0} \frac{-2xh + 3h - h^2}{h} \\&= \lim_{h \rightarrow 0} \frac{h(-2x + 3 - h)}{h} \\&= -2x + 3 \\f'(3) &= -3\end{aligned}$$

5. Find an equation of the tangent line to  $f(x) = -x^2 + 3x + 2$  at the point  $(3, 2)$ .

$$f'(x) = -2x + 3$$

$$m = f'(3) = -3$$

$$y = -3(x - 3) + 2$$

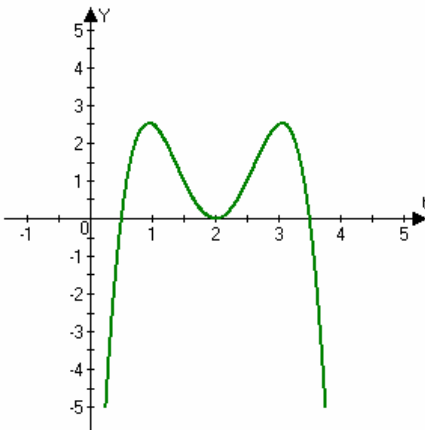
$$y = -3x + 11$$

6. Use the definition of the derivative to find  $f'(x)$  where  $f(x) = \sqrt{x+1}$ .

$$\begin{aligned}f(x) &= \sqrt{x+1} \\f'(x) &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\&= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \left( \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}} \right) \\&= \lim_{h \rightarrow 0} \frac{x+h+1 - x-1}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\&= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\&= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} \\&= \frac{1}{2(\sqrt{x+1})}\end{aligned}$$

7. The following is a graph of  $f'$ , the derivative of some polynomial function  $f$ .  
Use the graph to answer the following questions.

- a) On what interval(s) is  $f$  increasing?  $\left(\frac{1}{2}, 2\right) \cup \left(2, \frac{7}{2}\right)$
- b) On what interval(s) is  $f$  decreasing?  $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{7}{2}, \infty\right)$
- c) Where does  $f$  have a local minimum?  $\left(\frac{1}{2}\right)$
- d) Where does  $f$  have a local maximum?  $\left(\frac{7}{2}\right)$
- e) On what interval(s) is  $f$  concave up?  $(-\infty, 1) \cup (2, 3)$
- f) On what interval(s) is  $f$  concave down?  $(1, 2) \cup (3, \infty)$
- g) Does  $f$  have any inflection points? If yes, where are they? 1, 2, 3



8. Rules of Differentiation - Fill in the following formulas:

- a) The derivative of a constant function  $f(x) = k$  is 0
- b) The derivative of the function  $kf$  is  $(kf)' = kf'$
- c) The derivative of the sum function  $f + g$  is  $(f + g)' = f' + g'$
- d) The derivative of the difference function  $f - g$  is  $(f - g)' = f' - g'$
- e) The derivative of the product function  $fg$  is  $(fg)' = fg' + gf'$
- f) The derivative of the quotient function  $(f/g)$  is  $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

g) If  $n$  is any integer, then  $(x^n)' = nx^{n-1}$

h) The derivative of the sine function is  $(\sin x)' = \cos x$

i) The derivative of the cosine function is  $(\cos x)' = -\sin x$

j) The derivative of the secant function is  $(\sec x)' = \sec x \tan x$

k) The derivative of the cosecant function is  $(\csc x)' = -\csc x \cot x$

l) The derivative of the tangent function is  $(\tan x)' = \sec^2 x$

m) The derivative of the cotangent function is  $(\cot x)' = -\csc^2 x$

n) The derivative of the exponential function is  $(e^x)' = e^x$

8. Find the derivative,  $f'(x)$ , for each of the following functions:

a)  $f(x) = -2x^5 + 2x^{3/2} - \frac{3}{2}x^{-2} + x + \pi$        $f'(x) = -10x + 3x^{1/2} + 3x^{-3} + 1$

b)  $f(x) = 2 \cos x - 3 \sin x - 4e^x$        $f'(x) = -2 \sin x - 3 \cos x - 4e^x$

9. Find the derivative,  $f'(x)$ , for each of the following functions:

a)  $f(x) = x^3 \tan x$        $f'(x) = x^3 \sec^2 x + 3x^2 \tan x$

b)  $f(x) = (x^2 + x^{3/4})(\csc x - x^3)$

$$f'(x) = \left(x^2 + x^{3/4}\right)(-\csc x \cot x - 3x^2) + (\csc x - x^3)\left(2x + \frac{3}{4}x^{-1/4}\right)$$

c)  $f(x) = \frac{\sec x}{1-x^2}$        $f'(x) = \frac{(1-x^2)(\sec x \tan x) - (\sec x)(-2x)}{(1-x^2)^2}$

10. A particle moves according to the position function  $s(t) = 3t^3 - 18t + 4, t \geq 0$ , where  $t$  is measured in seconds and  $s$  is in meters. Use calculus to find the following exactly:

a) The velocity at time  $t$ :  $v(t) = s'(t) = 9t^2 - 18$

b) What is the velocity after 3s?

$$v(3) = s'(3) = 9(3^2) - 18$$

$$v(3) = s'(3) = 63m/s$$

c) When is the particle at rest?

The particle is at rest when:  $v(t) = s'(t) = 0$

That is, when  $9t^2 - 18 = 0$ . This means that  $t^2 = 2$  or  $t = \sqrt{2}$  seconds

d) When is the particle at moving forward?  $t > \sqrt{2}$  seconds

e) Find the acceleration at time  $t$ .  $a(t) = s''(t) = [s'(t)]' = 18t$

f) What is the acceleration after 3s?  $a(3) = s''(3) = 54m / \text{sec}^2$

11. Find the derivative,  $\frac{dy}{dx}$ , for the function:  $y = e^x(x - \cot x)$

$$\begin{aligned}\frac{dy}{dx} &= e^x(1 + \csc^2 x) + (x - \cot x)e^x \\ &= e^x[(1 - \csc x) + (x - \cot x)]\end{aligned}$$

a)  $f(x) = \frac{x}{1 - \sin x}$

$$\frac{dy}{dx} = \frac{(1 - \sin x)x' - x(1 - \sin x)'}{(1 - \sin x)^2}$$

$$= \frac{1 - \sin x + x \cos x}{(1 - \sin x)^2}$$

12. Let  $f(x) = \sqrt{2}x + 2 \cos x$ . Does  $f$  have any horizontal tangent lines in the interval  $-2\pi \leq x \leq 2\pi$ ? If so, where? If not, why not?

$$f'(x) = \sqrt{2} - 2 \sin x$$

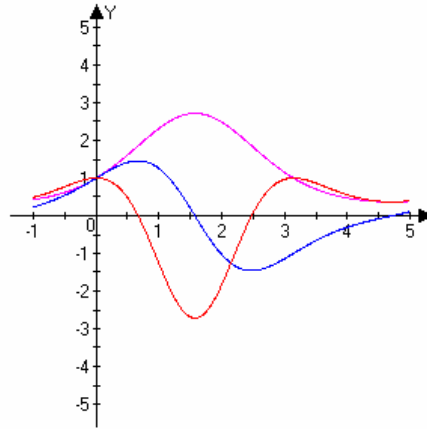
$$\sqrt{2} - 2 \sin x = 0$$

$$-2 \sin x = -\sqrt{2}$$

$$\sin x = \frac{\sqrt{2}}{2}$$

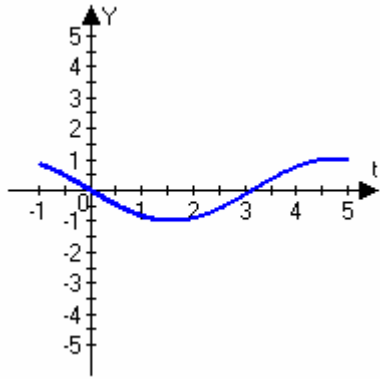
$$\text{Tangents at } \left( -\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4} \right)$$

13. The graph below contains the graphs of three functions -  $f, f',$  and  $f''$ . Place the correct label on each graph. pink= $f$ , blue= $f'$ , red= $f''$

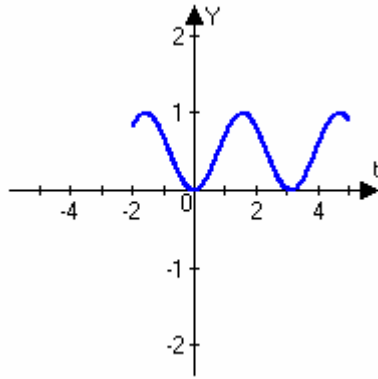


14. Find the derivative,  $\frac{dy}{dx}$ , for the function  $y = \frac{x}{1 - \sin x}$ .
15. For each of the functions A-F whose graphs are given on the next page, Match the function with its derivative function G-I. Place your answers here:

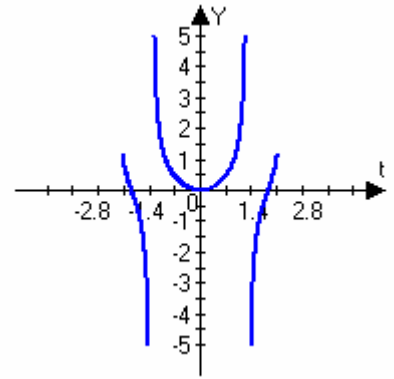
- a)  $A' = H$
- b)  $B' = I$
- c)  $C' = G$
- d)  $D' = K$
- e)  $E' = J$
- f)  $F' = L$



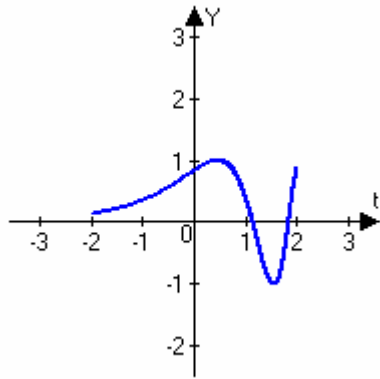
A



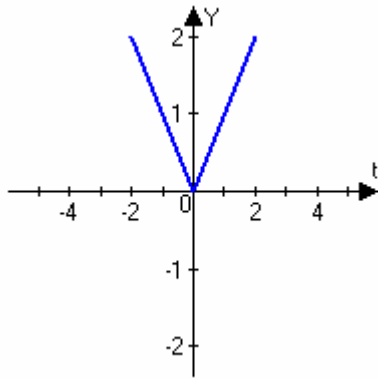
B



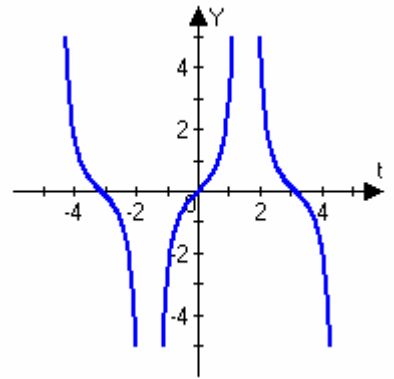
C



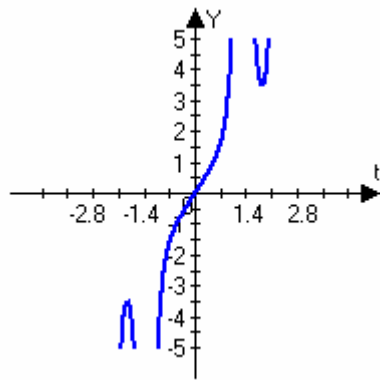
D



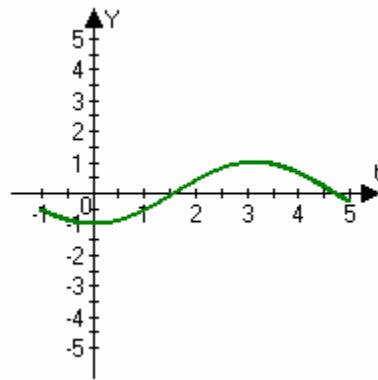
E



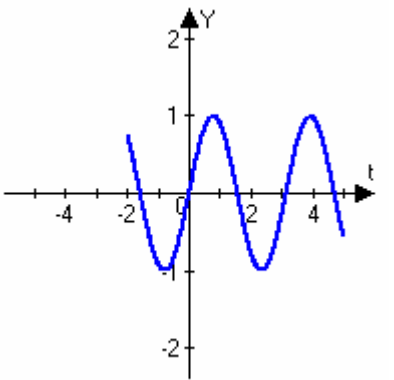
F



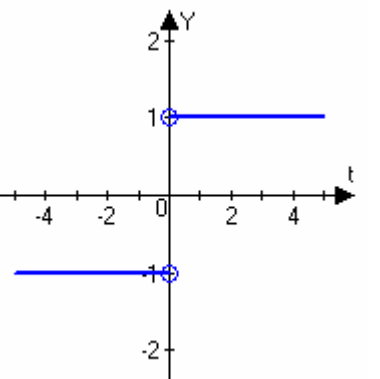
G



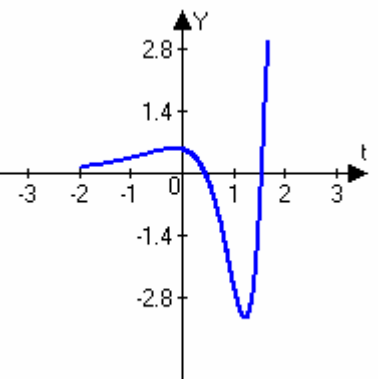
H



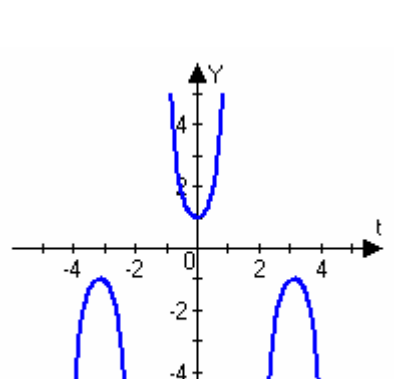
I



J



K



L

