

Name _____ *Calculus I Practice Final Exam*

Show all work on the test paper for partial credit.

1. Indefinite Integrals (Part 1)

a) $\int \left(\frac{6}{x} + \frac{2}{x^2} \right) dx$

b) $\int (\sin x + 3 \cos x) dx$

c) $\int \frac{1}{2} (e^x + e^{-x}) dx$

2. Indefinite Integrals:

a) $\int e^x (1 + e^x)^3 dx$

b) $\int x \sqrt{x^2 - 3} dx$

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3. Approximating Sums

Suppose you wish to approximate the area under the graph of $f(x) = e^{-x}$ for $0 \leq x \leq 1$. Approximate the area using eight approximating intervals ($n = 8$) and taking sample points to be midpoints

4. Interpretations of Integrals

a) Determine the position function if the acceleration function is $a(t) = 5$ with initial velocity $v(0) = 8$ and initial position is $s(0) = 0$.

b) Use the velocity function $v(t) = 20e^{-t/2}$ to compute the distance traveled over the time interval $[0, 2]$.

5. Definite Integrals (Part 1):

a) Find the value of the integral $\int_0^2 (x^2 - 2) dx$

b) Find the value of the integral $\int_0^{\pi/4} \sec^2 x dx$.

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c) Find the value of the integral $\int_0^1 x e^{-x^2} dx$.

d) Find the value of the integral $\int_0^{\pi/3} \sec x \tan x (1 + \sec x) dx$.

6. Fundamental Theorem:

a. Let $f(x) = \int_2^x (\sin t^3 - 2) dt$. Find the value of $f'(x)$.

b. Let $g(x) = \int_2^{5x} e^t dt$. Find the value of $g'(1)$

7. Definite Integrals (Part 2)

a) Find the value of the integral $\int_e^{e^2} \frac{\ln x}{x} dx$.

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b) Find the value of the integral $\int_0^1 \frac{e^x}{e^x + 1} dx$.

8. Riemann Sums

Evaluate the integral by computing the limit of Riemann sums.

$$\int_0^1 3x^2 dx$$

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9. Suppose you wish to approximate the area under the graph of $f(x) = x^2$ for $0 \leq x \leq 2$. What is the value of the estimate using four approximating intervals and taking sample points to be midpoints?

10. Rules of Definite Integrals:

a) If $\int_0^3 f(x)dx = 4$, $\int_3^6 f(x)dx = 4$ and $\int_2^6 f(x)dx = 5$, find the value of $\int_0^2 f(x)dx$.

b) If $\int_0^3 f(x)dx = 12$ and $\int_0^6 f(x)dx = 42$, find the value of $\int_3^6 [2f(x) - 3]dx$.

11. Definite Integrals (Part 1):

a) Find the value of the integral $\int_1^2 \frac{1}{x^2} dx$.

b) Find the value of the integral $\int_{-1}^8 \sqrt[3]{x} dx$.

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12. Fundamental Theorem:

a) Let $f(x) = \int_x^{10} t^3 dt$. Find the value of $f'(2)$

b) Let $g(x) = \int_2^{3x} e^t dt$. Find the value of $g'(0)$

13. Definite Integrals (Part 2):

a) Find the value of the integral $\int_0^{\pi/4} \cos 2x dx$

b) Find the value of the integral $\int_0^{\pi/3} \sec x \tan x (1 + \sec x) dx$

14. Indefinite Integrals (Part 1):

a) Find the value of the integral $\int_e^{e^2} \frac{1}{x \ln x} dx$

b) Find the value of the integral $\int_0^1 \frac{e^x}{1 + e^{2x}} dx$

15. Indefinite Integrals (Part 2):

a) Find the value of the integral $\int (x^2 + 1)^5 dx$

b) Find the value of the integral $\int \frac{x^2}{(x^3 + 1)^2} dx$

16. Indefinite Integrals (Part 3):

a) Find the value of the integral. $\int x e^{x^2} dx$

b) Find the value of the integral. $\int e^x \sin(e^x) dx$

17. Indefinite Integrals (Part 3):

a) Find the value of the integral. $\int (6x - 5) dx$

b) Find the value of the integral. $\int \left(\frac{4}{x} - \frac{3}{x^2} \right) dx$

18. Express $\lim_{n \rightarrow \infty} \left\{ \sum_{k=1}^n \left[5 + \left(\frac{3k}{n} \right)^2 \right] \frac{3}{n} \right\}$ as a definite integral

Answers:

1. a) $6\ln|x| - \frac{2}{x} + C$ b) $3\sin x - \cos x + C$ c) $\frac{1}{2}(e^x - e^{-x}) + C$

2. a) $\frac{1}{4}(1 + e^x)^4 + C$ b) $\frac{1}{3}(x^2 - 3)^{3/2} + C$

3. $A_8 = 0.6317$

4. a) $s(t) = \frac{5}{2}t^2 + 8t$ b) distance traveled is $40[1 - e^{-1}]$

5. a) $-\frac{4}{3}$ b) 1 c) $\frac{1}{2}[1 - e^{-1}]$ d) $\frac{5}{2}$

6. a) $\sin x^3 - 2$ b) $g'(1) = 5e^{25}$

7. a) $\frac{3}{2}$ b) $\ln\left(\frac{e+1}{2}\right)$

8. 1

9. 2.625

10. a) $\int_0^2 f(x) dx = 3$ b) 51

11. a) $\frac{1}{2}$ b) $\frac{45}{4}$

12. a) -8 b) 3

13. a) $\frac{1}{2}$ b) $\frac{5}{2}$

14. a) $\ln 2$ b) $\tan^{-1}(e) - \frac{\pi}{4}$

15. a) $\frac{1}{11}x^{11} + \frac{5}{9}x^9 + \frac{10}{7}x^7 + 3x^5 + \frac{5}{3}x^3 + x + C$ b) $\frac{-1}{3(x^3 + 1)} + C$

16. a) $\frac{1}{2}e^{x^2} + C$ b) $-\cos(e^x) + C$

17. a) $3x^2 - 5x + C$ b) $4\ln|x| + \frac{3}{x} + C$

18. $\int_0^3 (5 + x^2) dx$