**Name** ______________________________

# Calculus 1
# Final Exam
# Mike Huff
# Fall 2009

**Show all work on the test paper for partial credit.**

(4 points)

**1. Definitions/Short Answer**

a) State the Reimann sum definition of the definite integral $\int_a^b f(x)dx$ .

b) Carefully state both parts of the Fundamental Theorem of Calculus.

I.

II.

c) $\frac{d}{dx}\left(\int_a^b f(x)dx\right) = ?$

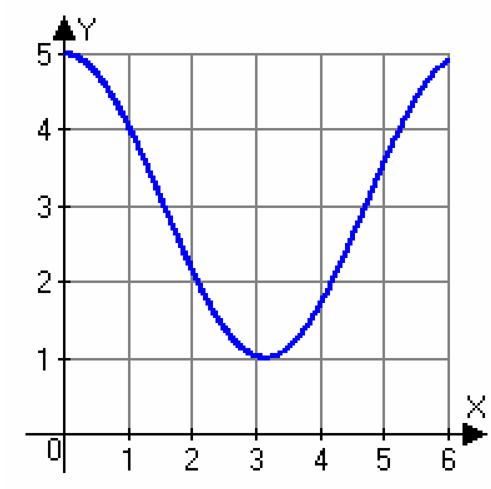
d) What does $\lim_{x \to c} f(x) = L$ mean? Sketch a graph below as part of your explanation.

**(6 points)**

**2. Approximating Sums**

Suppose you wish to approximate the area under the graph of $f(x) = 2\cos(x) + 3$ for $0 \le x \le 6$. The area is sketched below.

a) Divide the area into 6 rectangles below using the midpoint as the sample point of each rectangle. Sketch the 6 rectangles.




b) Approximate the area using six approximating intervals ( $n = 6$ ) and taking sample points to be ***midpoints***.

c) Compare your approximation to the exact value found using the Fundamental Theorem of Calculus.

**(12 points)**

3. **Indefinite Integrals (Part 1)**

**a)** $\int \left( x^{-9/5} - \frac{2}{x} + \sec^2 x \right) dx$

**b)** $\int \left( -\frac{3}{4} \cos x + \frac{5}{3} \sin x + e^x \right) dx$

**c)** $\int \left( 4 \csc x \cot x + \frac{3}{1 + x^2} \right) dx$

**(12 points)**

**4. Indefinite Integrals (Part 2)**

**a)** $\int e^x \sqrt{1+e^x}\, dx$

**b)** $\int x\sqrt{x+3}dx$

**c)** $\int \sin\left(5-\frac{\pi}{2}x\right)dx$

**(15 points)**

**5. Interpretations of Integrals**

a) Determine the position function, $s(t)$, of an object if the acceleration function of the object is $a(t) = -9.8\ \mathrm{m/s^2}$ the initial velocity $v(0) = 3.8\ \mathrm{m/s}$ and the initial position is $s(0) = 6.4$ .

b) At 10 am, water begins leaking from a tank at a rate of $2 + 0.55t$ gal/hour (t represents hours after 10 am). How much water is lost between 11 am and 3 pm?

c) Use the velocity function $v(t) = 2e^{-\frac{1}{2}t}$ to compute the distance traveled by an object over the time interval $[0, 2]$.

**(12 points)**

**6. Definite Integrals (Part 1):**

a) Find the value of the integral $\int_1^2 \left( \frac{1}{x} - x^3 \right) dx$

b) Find the value of the integral $\int_0^{\pi/6} \sec x \tan x dx$ .

c) Find the value of the integral $\int_{\pi}^{e^{\pi}} \frac{1}{x} dx$ .

(6 points)

**7. Fundamental Theorem:**

Let $f(x) = \int_{1}^{2x} \sqrt{\cos^2 t + t^3}\,dt$. Find the value of $f'(x)$.

**(12 points)**

**8. Definite Integrals (Part 3)**

**a)** Find the value of the integral $\int_{\pi/6}^{\pi/4} \sin x \cos^2 x dx$

**b)** Find the value of the integral $\int_{\pi/4}^{\pi/3} \tan x dx$

**(10 points)**

**9.** **Rules of Definite Integrals:**

a) $\int_{2}^{12} dx$

b) $\int_{0}^{8} x dx$

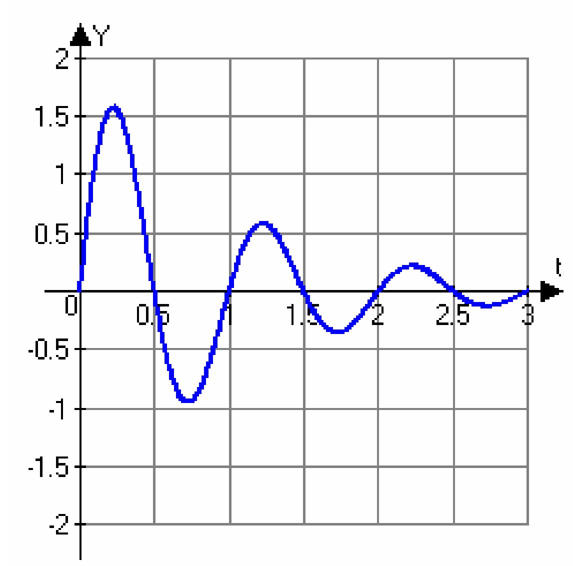
If $\int_{0}^{5} f(x)dx = 12$, $\int_{5}^{12} f(x)dx = 6$ and $\int_{8}^{12} f(x)dx = 4$, find the following:

c) $\int_{12}^{8} f(x)dx$

d) $\int_{0}^{8} \left[6f(x) - 4\right] dx$.

**(7 points)**

10. Let $g(x)=\int_0^x f(t)dt$ , where $f$ is the function whose graph is shown below.

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a. At what values of *x* do the local maximum and minimum values of *g* occur?

b. Where does *g* attain its absolute maximum value?

c. On what intervals is *g* concave downward?

**(4 points)**

11. An oil tanker ran aground and began to leak oil. Find the total amount of oil spilled between the first and second hours if the oil is leaking at a rate given by $f(t) = -\frac{1}{2}t^2 + 4$ thousands of gallons per hour, where *t* is time in hours since the spill. Determine when the leak stopped and approximately how much oil was carried.