

Answers for Review Problems for Calculus 1

1. $x \in \mathbb{R}, y \geq -\frac{4}{3}$

$$\begin{aligned} 3x^2 + 2x - 1 &= 0 \\ (3x-1)(x+1) &= 0 \\ 3x-1=0 \quad x+1=0 \\ x &= \frac{1}{3} \quad \text{or} \quad x = -1 \end{aligned}$$

2. $x \in \mathbb{R}, y \in \mathbb{R}$

$$\begin{aligned} 12t^3 + 12t^2 - 12t &= 0 \\ t(t^2 + t - 1) &= 0 \\ t=0 \quad \text{or} \quad t^2 + t - 1 &= 0 \\ t &= \frac{-1 \pm \sqrt{5}}{2} \end{aligned}$$

3. $x \in \mathbb{R}, -.125 \leq y \leq 1$

$$\begin{aligned} \frac{(r^2 + 1) - 2r^2}{(r^2 + 1)^2} &= 0 \\ r^2 - 2r^2 + 1 &= 0 \\ -r^2 &= -1 \\ r &= \pm 1 \end{aligned}$$

4. $x \in \mathbb{R}, -0.409 < y < 1.471$

$$\begin{aligned} \frac{x^2 + x + 1 - (x+1)(2x+1)}{(x^2 + x + 1)^2} &= 0 \\ x^2 + x + 1 - 2x^2 - 3x - 1 &= 0 \\ -x^2 - 2x &= 0 \\ -x(x-2) &= 0 \\ x=0 \quad \text{or} \quad x=2 \end{aligned}$$

5. $x \neq 0, y > -4.75$ or $y < -21.8$

$$\begin{aligned} \frac{(x-4)(14x-16)}{5x^5} &= 0 \\ x-4=0 \quad \text{or} \quad 14x-16=0 \\ x=4 \quad \text{or} \quad x &= \frac{8}{7} \end{aligned}$$

6. $x \neq 0, x \neq 1, y \in \mathbb{R}$

$$\begin{aligned} \frac{1}{3}(x^2 - x)^{-\frac{2}{3}}(2x-1) &= 0 \\ 2x-1 &= 0 \\ x &= \frac{1}{2} \end{aligned}$$

7. $x \in \mathbb{R}, -2 \leq y \leq 2$

$$\begin{aligned} 2\sin(2\theta)\cos(2\theta) &= 0 \\ \sin 2\theta = 0 \quad \text{or} \quad \cos 2\theta &= 0 \\ \text{For } k \in \mathbb{Z} \\ 2\theta_1 = 0 + 2k\pi \quad \text{or} \quad 2\theta_3 &= \frac{\pi}{2} + 2k\pi \\ 2\theta_2 = \pi + 2k\pi \quad \text{or} \quad 2\theta_4 &= \frac{3\pi}{2} + 2k\pi \\ \theta_1 = k\pi \\ \theta_2 = \frac{\pi}{2} + k\pi \\ \theta_3 = \frac{\pi}{4} + k\pi \\ \theta_4 = \frac{3\pi}{4} + k\pi \end{aligned}$$

8. $x > 0, y \in \mathbb{R}$

$$\begin{aligned} x\left(\frac{1}{x}\right) + \ln x &= 0 \\ \ln x &= -1 \\ x &= e^{-1} \approx 0.3679 \end{aligned}$$

9. $x \in \mathbb{R}, y > -0.135$

$$\begin{aligned} x(2e^{2x}) + e^{2x} &= 0 \\ e^{2x}(2x+1) &= 0 \\ 2x+1 &= 0 \\ x &= -\frac{1}{2} \end{aligned}$$

10. $-3 \leq x \leq 3, y \in \mathbb{R}, \frac{-x}{\sqrt{9-x^2}} = 0$
 $x = 0$

11. $x \neq 0, y \in \mathbb{Z}$

$$2x - \frac{2}{x^2} = 0$$

$$2x^3 - 2 = 0$$

$$x^3 - 1 = 0$$

$$x^3 = 1$$

$$x = 1$$

12. $x \in \mathbb{R}, y \leq 2$

$$\frac{(x^2 + 4) - 2x^2}{(x^2 + 4)2} = 0$$

$$x^2 + 4 - 2x^2 = 0$$

$$-x^2 + 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

13. $x \in \mathbb{R}, -1.4143 < y < 1.4143$

$$\cos x - \sin x = 0$$

$$\cos x = \sin x$$

$$\tan x = 1$$

$$x = \frac{\pi}{4} + k\pi$$

14. $x \in \mathbb{R}, -1 \leq y \leq 3$

$$1 + 2\sin x = 0$$

$$\sin x = -\frac{1}{2}$$

Reference angle $\frac{\pi}{6}$

$$Q3: x = \pi + \frac{\pi}{6} + 2k\pi = \frac{7\pi}{6} + 2k\pi$$

$$Q4: x = 2\pi - \frac{\pi}{6} + 2k\pi = \frac{11\pi}{6} + 2k\pi$$

15. $x \in \mathbb{R}, y \leq 1$

$$x(e^{-x}) + e^{-x} = 0$$

$$e^{-x}(x+1) = 0$$

$$x+1 = 0$$

$$x = -1$$

16. $x > 0, y \in \mathbb{R}$

$$\frac{x\left(\frac{1}{x}\right) - \ln x}{x^2} = 0$$

$$1 - \ln x = 0$$

$$\ln x = 1$$

$$x = e \approx 2.71828$$

17. $x \in \mathbb{R}, y > -1.08$

$$e^{x^3-x}(3x^2-1) = 0$$

$$3x^2 - 1 = 0$$

$$x^2 = \frac{1}{3}$$

$$x = \pm \sqrt{\frac{1}{3}} = \pm \frac{\sqrt{3}}{3}$$

18. $0 < x < 1, y < 0.59$

$$x \frac{1-2x}{2\sqrt{x-x^2}} + \sqrt{x-x^2} = 0$$

$$x(1-2x) + 2(x-x^2) = 0$$

$$x - 2x^2 + 2x - 2x^2 = 0$$

$$-4x^2 + 3x = 0$$

$$-x(4x-3) = 0$$

$$x = 0 \text{ or } x = \frac{3}{4}$$

extraneous

19. $x \in \mathbb{R}, -\frac{1}{3} \leq y \leq 1$

$$\frac{(2 + \sin x)(-\sin x) - (\cos x)(\cos x)}{(2 + \sin x)^2} = 0$$

$$-2\sin x - \sin^2 x - \cos^2 x = 0$$

$$-\sin^2 x - 2\sin x - (1 - \sin^2 x) = 0$$

$$2\sin x + 1 = 0$$

$$\sin x = -\frac{1}{2}$$

See #14 solution

$$8 - 8x^7 = 0$$

20. $x \in \mathbb{R}, y \notin \mathbb{R}$ $1 - x^7 = 0$

$$x^7 = 1$$

$$x = 1$$

21. $x \in \mathbb{R}, -1 \leq y \leq 3,$

$$1 - 2\cos x = 0$$

$$\cos x = \frac{1}{2}$$

$$x = \pm \frac{\pi}{3} + 2k\pi$$

22. $x \in \mathbb{R}, -2 \leq y \leq 2$

$$2\sin x = 0$$

$$\sin x = 0$$

$$x_1 = 0 + 2k\pi$$

$$x_2 = \pi + 2k\pi$$

or

$$x = \pi + k\pi$$

23. $x \neq -1, y \in \mathbb{R}$

$$\frac{(1+x)^2 - 2x(1+x)}{[(1+x)^2]^2} = 0$$

$$1 + 2x + x^2 - 2x - 2x^2 = 0$$

$$-x^2 + 1 = 0$$

$$x = 1 \text{ or } x = -1$$

extraneous

24. $x \neq -1, y < 0.008$

$$\frac{(1+x)^3(-1) - 3(1-x)(1+x)^2}{[(1+x)^3]^2} = 0$$

$$-(1+3x+3x^2+x^3) - 3(1-x)(1+2x+x^2) = 0$$

$$(-1-3x-3x^2-x^3) - 3(1+2x+x^2-x-2x^2-x^3) = 0$$

$$(-1-3x-3x^2-x^3) - 3(1+x-x^2-x^3) = 0$$

$$-1-3x-3x^2-x^3 - 3-3x+3x^2+3x^3 = 0$$

$$-4-6x+2x^3 = 0$$

From graph of $y = 2x^3 - 6x - 4$: $x = -1$ or $x = 2$
extraneous

25. $x \in \mathbb{R}, y > -0.135$

$$xe^x + e^x = 0$$

$$e^x(x+1) = 0$$

$$x+1 = 0$$

$$x = -1$$

26. $x \in \mathbb{R}, y > -0.05$

$$e^x + (x+1)e^x = 0$$

$$e^x(1+x+1) = 0$$

$$e^x(x+2) = 0$$

$$x+2 = 0$$

$$x = -2$$

27. $x \in \mathbb{R}, y > -0.462$

$$x^2e^x + 2xe^x = 0$$

$$e^x(x^2 + 2x) = 0$$

$$x^2 + 2x = 0$$

$$x(x+2) = 0$$

$$x = 0 \text{ or } x = -2$$

28. $x \in \mathbb{R}, y > -0.42$

$$(x^2 + 2x)e^x + e^x(2x + 2) = 0$$

$$e^x(x^2 + 2x + 2x + 2) = 0$$

$$e^x(x^2 + 4x + 2) = 0$$

$$x^2 + 4x + 2 = 0$$

$$x = \frac{-4 \pm \sqrt{16-8}}{2}$$

$$= -2 \pm \sqrt{2}$$

29. $x > 0, y > -0.0061$

$$\frac{\sqrt{x}\left(\frac{1}{x}\right) - \ln x \left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{x} = 0$$

$$\frac{1}{\sqrt{x}} - \frac{\ln x}{2\sqrt{x}} = 0$$

$$2 - \ln x = 0$$

$$\ln x = 2$$

$$x = e^2$$

30. $x > 0$

$$\frac{2x^{\frac{3}{2}}\left(-\frac{1}{x}\right) - (2 - \ln x)\left(3x^{\frac{1}{2}}\right)}{\left(2x^{\frac{3}{2}}\right)^2} = 0$$

$$-2x^{1/2} - 6x^{1/2} + 3x^{1/2} \ln x = 0$$

$$x^{1/2}(-2 - 6 + 3 \ln x) = 0$$

$$x^{1/2} = 0 \text{ or } 3 \ln x = 8$$

$$x = 0 \text{ or } \ln x = \frac{8}{3} \Rightarrow x = e^{8/3}$$

extraneous

31. $x > 0$

$$x\left(\frac{1}{x}\right) + \ln x = 0$$

$$1 + \ln x = 0$$

$$\ln x = -1$$

$$x = e^{-1}$$

32. $x < 1$

$$-\frac{1}{2}\left(-\frac{1}{2}\right)(1-x)^{-\frac{3}{2}}(-1) = 0$$

$$\frac{-1}{4(1-x)^{3/2}} = 0$$

no solution

33. $x \in \mathbb{R}$

$$x^4 \cdot 3(x-1)^2 + (x-1)^3 \cdot 4x^3 = 0$$

$$x^3(x-1)^2(7x+4) = 0$$

$$x = 0 \text{ or } x = 1 \text{ or } x = \frac{4}{7}$$

34. $x \in \mathbb{R}$

$$24x^2 + 4x^3 = 0$$

$$4x^2(6+x) = 0$$

$$x = 0 \text{ or } x = -6$$

35. $x \in \mathbb{R}$

$$15x^4 - 15x^2 = 0$$

$$15x^2(x^2 - 1) = 0$$

$$x = 0 \text{ or } x = \pm 1$$

36. $x \in \mathbb{R}$

$$60x^3 - 30x = 0$$

$$30x(2x^2 - 1) = 0$$

$$x = 0 \text{ or } x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$$

37. $x \neq 0$

$$1 - \frac{1}{x^{2/3}} = 0$$

$$x^{2/3} - 1 = 0$$

$$x^{2/3} = 1$$

$$x = 1^{3/2} = 1$$

38. $x \neq 0$

$$\frac{2}{3}x^{\frac{5}{3}} = 0$$

$$\frac{2}{3x^{5/3}} = 0$$

no solution

39. $x < 5$

$$x \cdot \frac{1}{2}(5-x)^{-\frac{1}{2}}(-1) + (5-x)^{\frac{1}{2}} = 0$$

$$\frac{-x}{\sqrt{5-x}} + \sqrt{5-x} = 0$$

$$-x + 2(5-x) = 0$$

$$10 - 3x = 0$$

$$x = \frac{10}{3}$$

40. $x \neq k\pi$

$$2 - \csc^2 x = 0$$

$$2 - \frac{1}{\sin^2 x} = 0$$

$$2\sin^2 x - 1 = 0$$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$$

$$Q1: x = \frac{\pi}{4} + 2k\pi$$

$$Q2: x = \frac{3\pi}{4} + 2k\pi$$

$$Q3: x = \frac{5\pi}{4} + 2k\pi$$

$$Q4: x = \frac{7\pi}{4} + 2k\pi$$

41. $x \neq k\pi$

$$-2\csc x(-\csc x \cot x) = 0$$

$$\frac{-2}{\sin x} \left(\frac{-1}{\sin x} \right) \left(\frac{\cos x}{\sin x} \right) = 0$$

$$\frac{2\cos x}{\sin^3 x} = 0$$

$$\cos x = 0$$

$$x = \pm \frac{\pi}{2} + 2k\pi$$

42. $x \in \mathbb{R}$

$$-2\sin x + 2\sin x \cos x = 0$$

$$2\sin x(-1 + \cos x) = 0$$

$$2\sin x = 0 \text{ or } \cos x - 1 = 0$$

$$x = k\pi \text{ or } \cos x = 1$$

$$x = 2k\pi$$

$$x = k\pi \text{ describes all}$$

43. $x \in \mathbb{R}$

$$-2\cos x + 2\cos 2x = 0$$

$$-2\cos x + 2(2\cos^2 x - 1) = 0$$

$$-2\cos x + 4\cos^2 x - 2 = 0$$

$$4\cos^2 x - 2\cos x - 2 = 0$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$$\cos x = -\frac{1}{2} \text{ or } \cos x = 1$$

$$x = \pm \frac{2\pi}{3} + 2k\pi \text{ or } x = 2k\pi$$

44. $x \in \mathbb{R}$

$$\frac{2x}{1+x^2} = 0$$

$$2x = 0$$

$$x = 0$$

45. $x \in \mathbb{R}$

$$\frac{(1+x^2)(2) - 2x(2x)}{(1+x^2)^2} = 0$$

$$2 + 2x^2 - 4x^2 = 0$$

$$2 - 2x^2 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

46. $x \neq \pm 1$

$$\frac{(1-x^2)(2x) - (1-x^2)(-2x)}{(1-x^2)^2} = 0$$

$$2x - 2x^3 + 2x - 2x^3 = 0$$

$$-4x^3 + 4x = 0$$

$$-4x(x^2 - 1) = 0$$

$$x = 0 \text{ or } x = \pm 1$$

both extraneous

47. $x \neq \pm 1$

$$\frac{(1-x^2)^2 \cdot 4 - 4x \cdot 2(1-x^2)(-2x)}{\left[(1-x^2)^2 \right]^2} = 0$$

$$4(1-2x^2+x^4) + 16x^2(1-x^2) = 0$$

$$4 - 8x^2 + 4x^4 + 16x^2 - 16x^4 = 0$$

$$-12x^4 + 8x^2 + 4 = 0$$

$$3x^4 - 2x^2 - 1 = 0$$

$$(3x^2 + 1)(x^2 - 1) = 0$$

$$3x^2 + 1 = 0 \text{ or } x^2 - 1 = 0$$

$$\text{No solution} \quad x = \pm 1 (\text{not in the domain})$$

$$\text{No Solution}$$

48. $x \neq 1$

$$\frac{(x-1)^2(1) - x(2)(x-1)}{\left[(x-1)^2 \right]^2} = 0$$

$$(x^2 - 2x + 1) - (2x^2 - 2x) = 0$$

$$x^2 - 2x + 1 - 2x^2 + 2x = 0$$

$$-x^2 + 1 = 0$$

$$x = 1 \text{ or } x = -1$$

extraneous

49. $x \in \mathbb{R}$

$$-\cos x(e^{\cos x}) - \sin x(e^{\cos x})(-\sin x) = 0$$

$$e^{\cos x}(-\cos x + \sin^2 x) = 0$$

$$-\cos x + (1 - \cos^2 x) = 0$$

$$-\cos^2 x - \cos x + 1 = 0$$

$$\cos^2 x + \cos x - 1 = 0$$

$$\cos x = \frac{-1 \pm \sqrt{5}}{2}$$

$$\cos x_1 = \frac{-1 - \sqrt{5}}{2} \approx -1.618$$

no \mathbb{R} solution for x_1

$$\cos x = \frac{-1 + \sqrt{5}}{2} \approx 0.6180$$

$$x \approx 0.9046 + 2k\pi$$

50. $x \in \mathbb{R}$

$$x^2(x-1)^2(7x-4) + x^3 \cdot 2(x-1)(7x-4) + x^3(x-1)^2 \cdot 7 = 0$$

$$x^2(x-1)[(x-1)(7x-4) + 2x(7x-4) + 7x(x-1)] = 0$$

$$x^2(x-1)[(7x^2 - 11x + 4) + (14x^2 - 8x) + (7x^2 - 7x)] = 0$$

$$x^2(x-1)[28x^2 - 26x + 4] = 0$$

$$x = 0 \text{ or } x = 1 \text{ or } x = \frac{26 \pm \sqrt{676 - 448}}{56} = \frac{26 \pm \sqrt{228}}{56}$$

51. $x < 5$

$$\frac{2(5-x)^{\frac{1}{2}}(-3) - (10-3x) \cdot 2\left(\frac{1}{2}\right)(5-x)^{-\frac{1}{2}}(-1)}{(2\sqrt{5-x})^2} = 0$$

$$(5-x)^{-1/2}[-6(5-x) + (10-3x)] = 0$$

$$-30 + 6x + 10 - 3x = 0$$

$$3x - 20 = 0$$

$$x = \frac{20}{3}$$

extraneous

No solution