

## Antiderivatives

**Definition:** An **antiderivative** of the function  $f$  is any function  $F$  for which  $F' = f$ .  
An antiderivative of the function  $f$  is denoted  $\int f(x)dx$ .

**Theorem 1:** If  $F$  is an **antiderivative** of the function  $f$  on an interval  $I$ , then the most general antiderivative of  $f$  on  $I$  is  $F(x) + C$  where  $C$  is any constant.

### Example 1: Finding Antiderivatives

a)  $f(x) = 6x^3 - 5x^2 + 1$

b)  $f(x) = e^x + \sec^2 x - 1$

c)  $f(x) = \frac{x^3 - x^2 + x - 1}{x}$

**Example 2: Finding antiderivatives satisfying a given condition**

Find the antiderivative of  $f(x) = 4 - 3(1 + x^2)^{-1}$  that satisfies,  $F(0) = 1$

**Example 3: Finding  $f$  given the second derivative**

Find  $f$  given  $f''(x) = \cos x$

**Example 4: Finding  $f$  given the second derivative**

Find  $f$  given  $f''(x) = x + \sqrt{x}$  ,  $f(1) = 1$  , and  $f'(1) = 2$  .

### Properties of the antiderivative:

1. **Theorem 1:** If  $F$  and  $G$  are two antiderivatives of  $f$  on an interval  $[a, b]$ , then there is a constant  $c$ , such that  $F(x) = G(x) + c$
2. **Theorem 2:** If  $f$  and  $g$  are two functions with antiderivatives  $\int f(x)dx$  and  $\int g(x)dx$ , then the following hold:
  - a)  $\int cf(x)dx = c \int f(x)dx$  for any constant  $c$ .
  - b)  $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$ .
  - c)  $\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$ .

### Application

If the acceleration of an object at any time is known and the initial values of the objects position,  $s(0)$ , and its initial velocity,  $v(0)$ , are known, then the objects position at any time can be found. This is true since:

If $s(t)$ is given then,	If $a(t)$ is given then,
$v(t) = s'(t)$	$v(t) = at + v(0)$
$a(t) = v'(t)$	$s(t) = a \frac{t^2}{2} + v(0)t + s(0)$

### Example 5: Motion

A particle moves with acceleration function  $a(t) = 4 + 3t - t^2$ . Its initial velocity is  $v(0) = 2\text{m/s}$  and its initial displacement is  $s(0) = 5\text{m}$ . Finds its position after  $t$  seconds.