

Name _____

Calculus 1
MATH 2413
Project
Taylor
Polynomials

The tangent line approximation to a function at a point that we developed in class represents the best linear approximation to a curve at a point. In this project, you will develop higher-degree polynomial approximations that are called Taylor polynomials. In Calculus II, we will extend this idea to polynomials that have an infinite number of terms called Taylor series.

The linear approximation was created by finding a line with two characteristics: First, the function and the linear approximation had the same value at the indicated point, and second, the slope of the line was the same as the derivative of the function at the given point. If the point is $x = c$, then we want:

$$L(c) = f(c) \quad \text{Same value at } x = c$$

$$L'(c) = f'(c) \quad \text{Same slope at } x = c$$

To get a quadratic approximation, we will need to have three conditions. If the quadratic function will have the form $P(x) = c_1 + c_2x + c_3x^2$, then we must have

$$\text{i) } P(c) = f(c)$$

$$\text{ii) } P'(c) = f'(c)$$

$$\text{iii) } P''(c) = f''(c) \quad \text{The slopes change at same rate at } x = c$$

Exercise 1 Use $P(x) = a_0 + a_1x + a_2x^2$ to find a quadratic approximation to the function $f(x) = e^x$ at the point $c = 0$. Graph f , L , and P on the same coordinate grid below. How well do P and L approximate the function f ?

Exercise 2 Determine values of x for which the quadratic approximation to $f(x) = e^x$ in Exercise 1 is accurate to within 0.2. One way to do this is to graph $f(x) = e^x$, $P(x) - 0.2$, and $P(x) + 0.2$ on the same screen and find the points of intersection.

Exercise 3 Use $P(x) = a_0 + a_1x + a_2x^2$ to find a quadratic approximation to the function $f(x) = \cos(x)$ at the point $c = 0$. Graph f , L , and P on the same coordinate grid below. How well do P and L approximate the function f ?

Exercise 4 In general, if we wish to approximate function values near a number c with a quadratic approximation it is best to use a quadratic approximation of the form $P(x) = a_0 + a_1(x - c) + a_2(x - c)^2$. Find the coefficients $a_0, a_1,$ and, a_2 for this polynomial so that it satisfies the three conditions i), ii), and iii) listed above.

Exercise 5 Develop a formula for a cubic approximation of the form

$P(x) = a_0 + a_1(x - c) + a_2(x - c)^2 + a_3(x - c)^3$ that satisfies the three conditions i), ii), and iii) listed above and the additional condition $P'''(c) = f'''(c)$ for a function f .

Exercise 6 Use $P(x) = a_0 + a_1(x - c) + a_2(x - c)^2 + a_3(x - c)^3$ to find a cubic approximation to the function $f(x) = e^x$ at the point $c = 0$. Graph f , L , and P on the same coordinate grid below. How well do P and L approximate the function f ?