

Name _____ Calculus I Test 1 Fall 2002 Mike Huff

The Algebra of Functions (8 points)

1. Given $f(x) = x^2 + 5x - 3$ and $g(x) = \sqrt{3 - x^2}$, find each of the following.

a) $f(x + h)$

b) $[g(x)]^2$

c) The domain of g .

d) The range of f .

The Difference Quotient (6 points)

2. If $f(x) = x^2 + 2x - 5$, find and simplify $\frac{f(x + h) - f(x)}{h}$

Transformations (9 points)

3. Describe how the graph of each function can be obtained from the graph of the function $f(x)$.

a) $f(x + 3)$

b) $2 - 4f\left(\frac{x}{3}\right)$

c) $f^{-1}(x)$

Inverse Functions (11 points)

4. For the function f given by $f(x) = \frac{2x + 3}{x - 1}$. Find:

a) The inverse function $f^{-1}(x)$.

b) The domain of f .

c) The range of f .

d) The domain of f^{-1} .

e) The range of f^{-1} .

Algebra of Functions (9 points)

5. If $f(x) = 3 - x^2$ and $g(x) = \sqrt{2 - x}$. Find the following functions and their domains:

a) $(f \circ g)(x)$

b) $(g \circ f)(x)$

c) $\left(\frac{f}{g}\right)(x)$

Solving Exponential Equations (8 points)

6. Solve for x exactly.

a) $e^{x^2+6} = e^{-5x}$

b) $e^{e^x} = 9$

Solving Logarithmic Equations (10 points)

7. Solve for x *exactly*.

a) $\ln(x + 3) + \ln x = 2 \ln 2$

b) $\ln(\log x) = e$

Word Problems (10 points)

- 8.
- a) A population of bacteria is known to grow exponentially. The size of the population is estimated by measuring the area of the culture dish that the colony covers. If initially the culture covers 4 square centimeters of the culture medium and after 3 days it covers 9 square centimeters, how much of the culture medium will be covered in 5 days?
- b) A metal container, in the form of a rectangular solid, is to be constructed. The base is to be square, there is to be no top, and the volume of the container is to be 12 cubic meters. Suppose the material for the sides cost \$2 per square meter, and the material for the base costs \$3 per square meter. Let C be the cost of the container, and L be the length of a side of the square base. Find a function giving the cost C in terms of L , and find its domain.

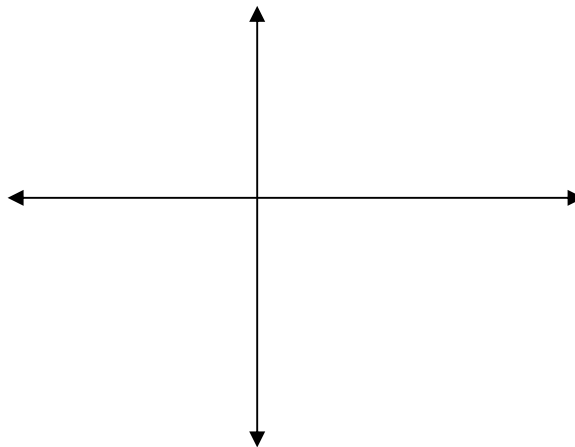
Graphing (10 points)

9. Sketch a graph of the following functions:

- a) $g(x) = 1 - 2 \cos\left(3x - \frac{\pi}{2}\right)$ (Draw exactly two periods and label the axes carefully.)



- b) $h(x) = \sin\left(\frac{1}{x}\right)$ What is the domain of this function? What is happening as x gets close to zero?



Assorted Problems (15 points)

10.

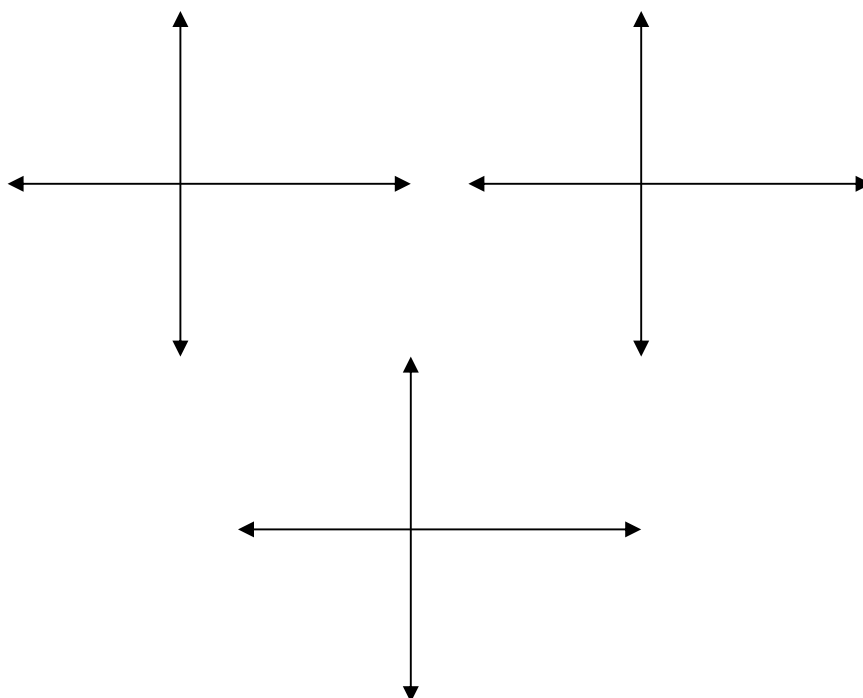
a) Express the area A of a circle as a function of its circumference.

b) Given the function $\sin x$ with domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, find the domain of its inverse.

c) For what value of x is $2^x = \left(\frac{1}{2}\right)^x$?

d) Determine the number of solutions of the equation $8 \sin 2x - 1 = x$

e) Sketch $f^{-1}(x)$ and $-f^{-1}(x)$ for the function f whose graph is drawn below.



(4 points)

11. Solve the following inequality:

$$2 < \ln x \leq 5$$