

Name \_\_\_\_\_

**Calculus I**  
**MATH 2413**  
**Test 1B**  
**Fall 2008**  
**Mike Huff**

**The Difference Quotient (8 points)**

1. Let  $f(x) = -x^3 + x$  to find:

a. 
$$\frac{f(x+h) - f(x)}{h}$$

b. 
$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Solving Equations (6 points)**

2. Solve for  $x$  exactly.

a)  $e^{\sec x} - e = 0$

b)  $e^{x^3} = \ln 5$

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**Solving Equations (12 points)**

3. Solve for  $x$  *exactly*.

a)  $\log(x - 5) + \log(1 - x) = \log(x)$

b)  $\cos^2 x - 2 \cos x + 1 = 0$

c) Solve:  $\frac{(x+1)^2(2) - 2x(2)(x+1)}{[(x+1)^2]^2} = 0$

**Secant and Tangent Lines (8 points)**

4. Let  $f(x) = \cos x$ , to find the following:

a) The slope  $M_{PQ}$  of the secant line through the points  $P = (0, f(0))$  and

$$Q = \left( \frac{\pi}{3}, f\left(\frac{\pi}{3}\right) \right).$$

b) Use the result of a) to find an equation for the secant line that goes through the

points  $P = (0, f(0))$  and  $Q = \left( \frac{\pi}{3}, f\left(\frac{\pi}{3}\right) \right)$  for the function  $f(x) = \cos x$ .

**Average Velocity (9 points)**

5. Suppose the height of a projectile fired vertically upward from a height of 1,237 feet with an initial velocity of 555 feet per second is given by  $s(t) = -16t^2 + 555t + 1237$ .

a) What is the height of the object after 5 seconds?

b) Find the average velocity over the time period  $[3, 3.001]$ .

c) Find the instantaneous velocity at  $t = 3$  seconds.



**Limits and Continuity (9 points)**

7. Use the graph of the function  $f(x)$  below to answer the following questions:

a)  $\lim_{x \rightarrow -2^-} f(x)$

b)  $\lim_{x \rightarrow -2^+} f(x)$

c)  $\lim_{x \rightarrow -2} f(x)$

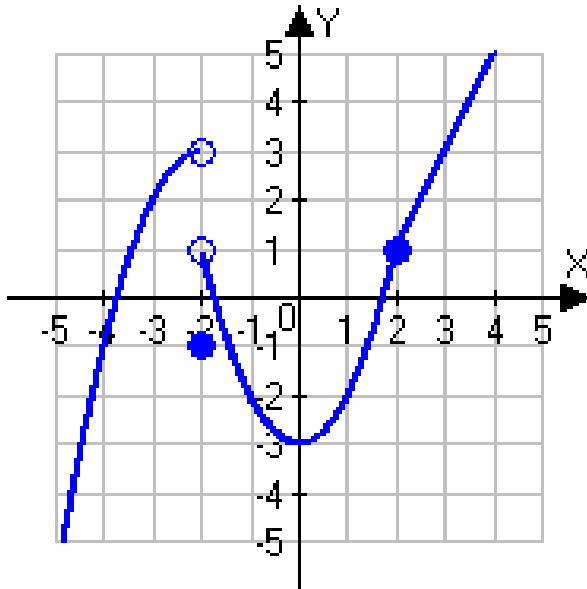
d)  $f(-2)$

e) Is  $f$  continuous at  $x = -2$ ? Explain.

f)  $\lim_{x \rightarrow 0} f(x)$

g)  $\lim_{x \rightarrow 2} f(x)$

h) Is  $f$  differentiable at  $x = 2$ ? Explain



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**Limits (6 points)**

8. Use algebra to find the value of the limit:  $\lim_{x \rightarrow 1} \left( \frac{x-1}{x^2+x-2} \right)$

**Limits (9 points)**

9. Find the value of the limits:

a)  $\lim_{x \rightarrow -\infty} \frac{x^3 - 2x^2 - x}{5 + x^2 - 3x^3}$

b)  $\lim_{x \rightarrow 3^+} \frac{(2x+3)(x+5)}{(x+2)(x-3)}$

c)  $\lim_{x \rightarrow 1^-} \frac{(x+3)(x+5)}{(x-1)(x+1)}$

**Limits (6 points)**

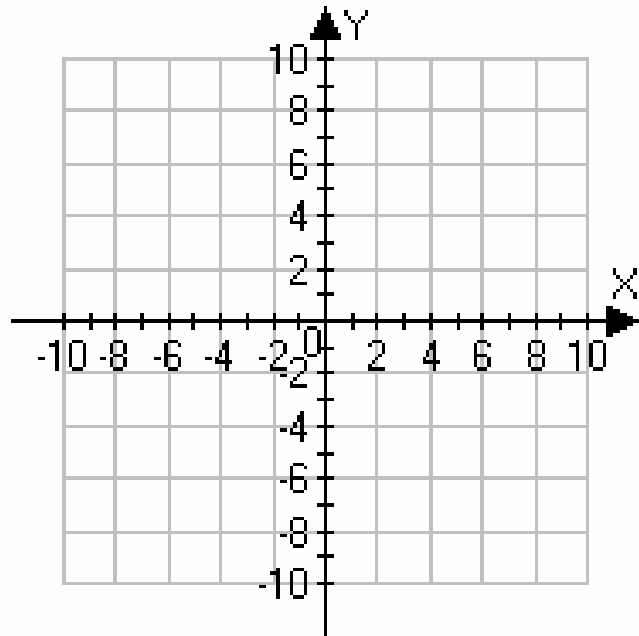
10. Find the value of the limit:  $\lim_{x \rightarrow -1} \sqrt{\frac{-\cos(\pi x)}{x^2 + x + 3}}$

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**Continuity (6 points)**

11. Sketch a graph of the function  $f(x) = \begin{cases} -2x^2 & x \leq 0 \\ 3 - 2x^2 & 0 < x < 1 \\ 4x - 3 & x \geq 1 \end{cases}$ . At what value or values

of  $x$  is the function discontinuous? **Explain carefully.**



**Limits at Infinity (6 points)**

12. Find the value of the limit:  $\lim_{x \rightarrow \infty} \left( \sqrt{(x+1)(x+5)} - x \right)$

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**Squeeze Theorem (6 points)**

13. Find  $\lim_{x \rightarrow \infty} f(x)$  if  $\tan^{-1} x \leq f(x) \leq \frac{3 - 8x + \pi x^2}{2x^2 - 4x + 11}$  for all  $x$ . Be careful to explain your reasoning.