

Name _____

Math 2413
Calculus 1
Mike Huff
Test 3
Fall 2009

Show all work on the test paper for partial credit.

(6 points)

1. **Rules of Differentiation:** Fill in the following formulas assuming b is a nonzero real number:

a) The derivative of the function $f(x) = e^e$ is $\frac{d}{dx}(e^e) =$

b) The derivative of the function $f(x) = b^x$ is $(b^x)' =$

c) The derivative of the natural logarithmic function is $(\ln x)' =$

d) The derivative of the function $f(x) = \log_b x$ is $(\log_b x)' =$

e) The derivative of the composite function $f(g(x))$ is $[f(g(x))]' =$

f) The derivative of the composite function $\ln(f(x))$ is $[\ln(f(x))]' =$

g) The derivative of the composite function $[f(x)]^n$ is $[(f(x))^n]' =$

h) The derivative of the composite function $\sin[f(x)]$ is $(\sin[f(x)])' =$

i) The derivative of the composite function $\cos[f(x)]$ is $(\cos[f(x)])' =$

j) The derivative of the arcsine function is $\frac{d}{dx}(\sin^{-1} x) =$

k) The derivative of the arctangent function is $\frac{d}{dx}(\tan^{-1} x) =$

l) The derivative of the composite function is $\frac{d}{dx}(e^{f(x)}) =$

Graphing and the Derivative

(10 points)

2. Given $f(x) = 4x + 2 - 5 \ln(1 + x^2)$, find the following:

a) The intervals where f is increasing _____

b) The intervals where f is decreasing. _____

c) Local maximum value(s) _____

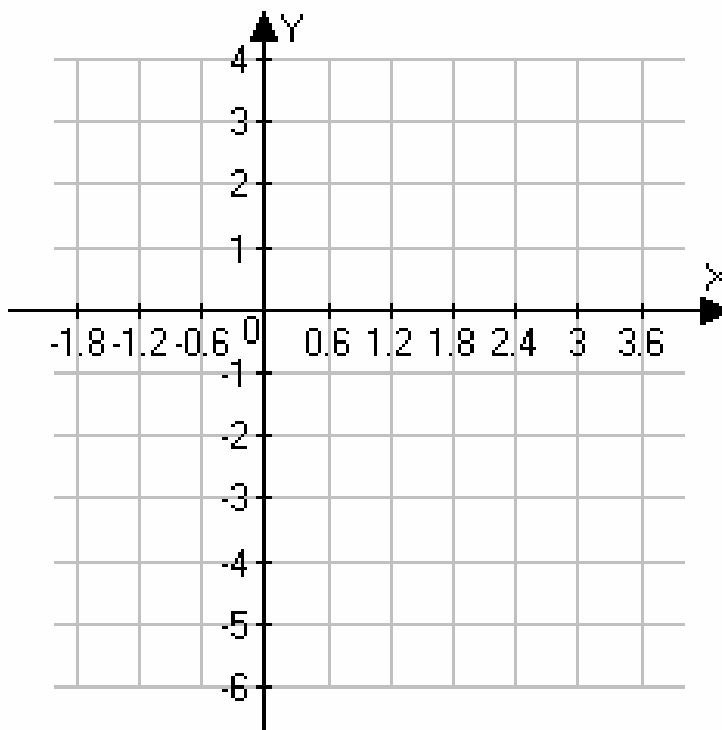
d) Local minimum value(s) _____

e) The intervals where f is concave up _____

f) The intervals where f is concave down. _____

g) Inflection point(s) _____

h) Sketch the graph.



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(16 points)

3. **Differentiation:**

Find the derivative of each of the following functions:

a) $f(x) = \cos^{-1}(x^2)$

b) $f(x) = \ln(5x^3 + x + \sqrt{x})$

c) $f(x) = 7^x$

d) $f(x) = \log_6 2x$

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(8 points)

4. Find the **differential** dy of each function.

a) $y = \tan^{-1}(7x)$

b) $y = \ln(\cos x)$

(6 points)

5. Calculate y' : $y = \frac{\sqrt{3x+1}(4x+1)^6}{(1-2x)^5}$

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(8 points)

6. Find the absolute maximum and absolute minimum of the function $f(x) = x^2e^{-x} + 1$ on the interval $[-2, 1]$.

(8 points)

7. Find the linear approximation to $f(x) = \sqrt[3]{x}$ at $a = 27$.
- a. Use it to approximate $\sqrt[3]{27.2}$.

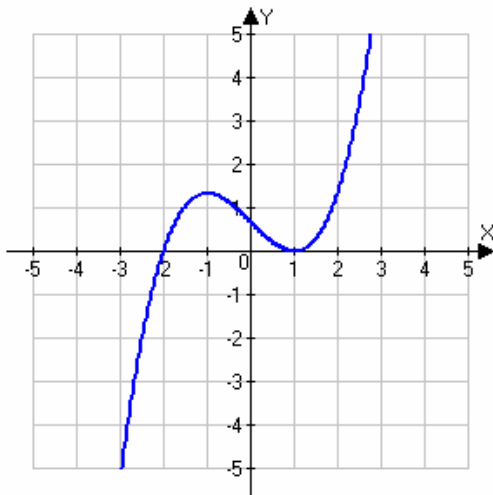
b. Use a differential to make the same approximation.

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(8 points)

8. Graphs and Derivatives

The given graph is the graph of f' ; the derivative of some function differentiable function f . Use the graph to find the following:



a) The intervals where is f increasing?

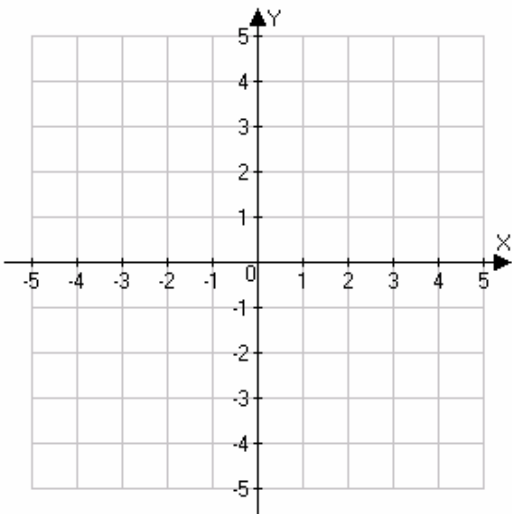
b) _____
The intervals where is f decreasing?

c) _____
The intervals where is f concave up?

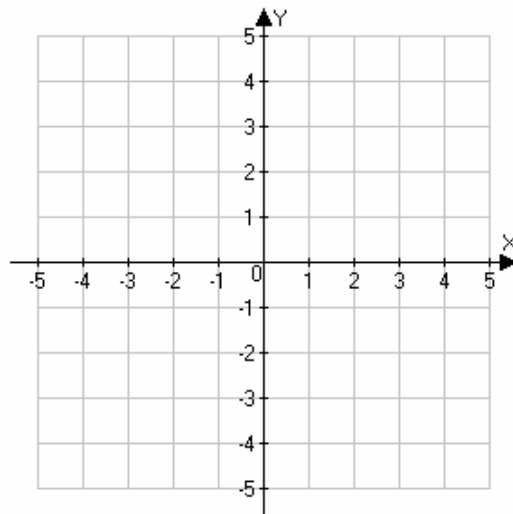
d) _____
The intervals where is f concave down?

e) _____
The points of inflection of f

f) _____
Sketch possible graphs of f and f'' below



f



f''

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(6 points)

9. Calculate y' for $y^2 + 2xy^2 = 3x - 2y$

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(12 points)

10. Related Rates

- a. The area of a circle is expanding at a rate of 10 cm^2 per second. How fast is the radius changing when the area is $\frac{1}{\pi} \text{ cm}^2$?
- b. One airplane is approaching an airport from the north at 206 km/hr . A second airplane approaches from the east at 211 km/hr . Find the rate at which the distance between the planes changes when the southbound plane is 21 km away from the airport and the westbound plane is 30 km from the airport.

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11. Applications (12 points – 4 points each)

- a. The population of a bacteria colony after t hours is given by $P(t) = 2000e^{0.087t}$. Find the growth rate of the colony when $t = 16$ hours.
- b. A tank is in the shape of a large, inverted (point-down) cone. Water is owing into the tank at a constant rate. Let $H(t)$ represent the height of the water level at time t .
- (a) Sketch a possible graph of $H(t)$.
- (b) Describe how the rate of change of H with respect to t varies as t increases.
- (c) Sketch a graph of $H'(t)$.
- c. The relationship between the rate of a certain chemical reaction and temperature under certain circumstances is given by $R(T) = 0.1(-0.05T^3 + 4T^2 + 120)$ grams/sec, where R is the rate of reaction and T is the temperature (in °C).
- (a) Find the temperature T at which the reaction rate reaches its maximum.
- (b) What is the maximum reaction rate?

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