Section 12 Probability

A continuous random variable is a variable whose values range over an interval of real numbers. Every continuous random variable X has a probability density function (or pdf). A probability density function is a function for which $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)dx = 1$. The probability that the variable X is between *a* and *b* is given by $P(a \leq X \leq b) = \int_a^b f(x)dx$.

Example 1: Using a Probability Density Function

Let $f(x) = \begin{cases} kx^2(1-x) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$ to answer the following.

a) For what values of *k* is *f* a probability density function?

b) For that value of *k*, find $P(X \geq \frac{1}{2})$

Definition: The mean of a pdf is defined to be $\mu = \int_{-\infty}^{\infty} xf(x)dx$

Example 2: Finding the Mean of a Probability Density Function

Find the mean of the probability density function $f(x)=\begin{cases}12x^2(1-x) & 0\le x\le 1\\ 0 & otherwise\end{cases}$.

Definition: The exponential probability density function is given by

$$f(x)=\begin{cases}o & x<0\\ \frac{1}{k}e^{-x/k} & x\ge 0\end{cases}.$$

Example 3: Finding the Mean of a Probability Density Function

Find the mean of the exponential probability density function $f(x)=\begin{cases}o & x<0\\ \frac{1}{k}e^{-x/k} & x\ge 0\end{cases}$.

Definition: The **median** of a probability density function is the number m such that $\int_{m}^{\infty} f(x)dx = \frac{1}{2}$

Example 4: Finding the Median of a Probability Density Function

Find the median of the exponential probability density function $f(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{k}e^{-x/k} & x \geq 0 \end{cases}$.

Example 5: Using a Probability Density Function

Suppose that the age in days of a single-celled organism has pdf $f(x) = (\ln 2)e^{-kx}$ where $k = \frac{1}{2}\ln 2$. The domain is $0 \leq x \leq 2$. The assumption here is that upon reaching an age of two days, each cell divides into two daughter cells.

Find:

a) The mean age (μ) of the cells. Hint: $\mu = \int_{-\infty}^{\infty} xf(x)dx$.

b) The proportion of cells that are younger than the mean.

c) The median age of the cells.