Section 3 More Integration Techniques

Trigonometric Integrals

If the integrand contains odd powers of sine or cosine, split off one and use substitution.

Example 1: Trigonometric Integrals with One Odd Power of Sine or Cosine

a) $\int \sin^3 x dx$

$= \int \sin^2 x \sin x dx$

$= \int \left(1-\cos^2 x\right) \sin x dx$ Now use the substitution $u = \cos x$.

b) $\int_0^{\pi/2} \cos^5 x dx$

c) $\int \sin^3 x \cos^2 x dx$

If the integrand contains even powers of sine or cosine, use the half-angle identities to simplify the integrand.

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

and

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

Example 2: Integrand Contains Even Powers of Sine or Cosine

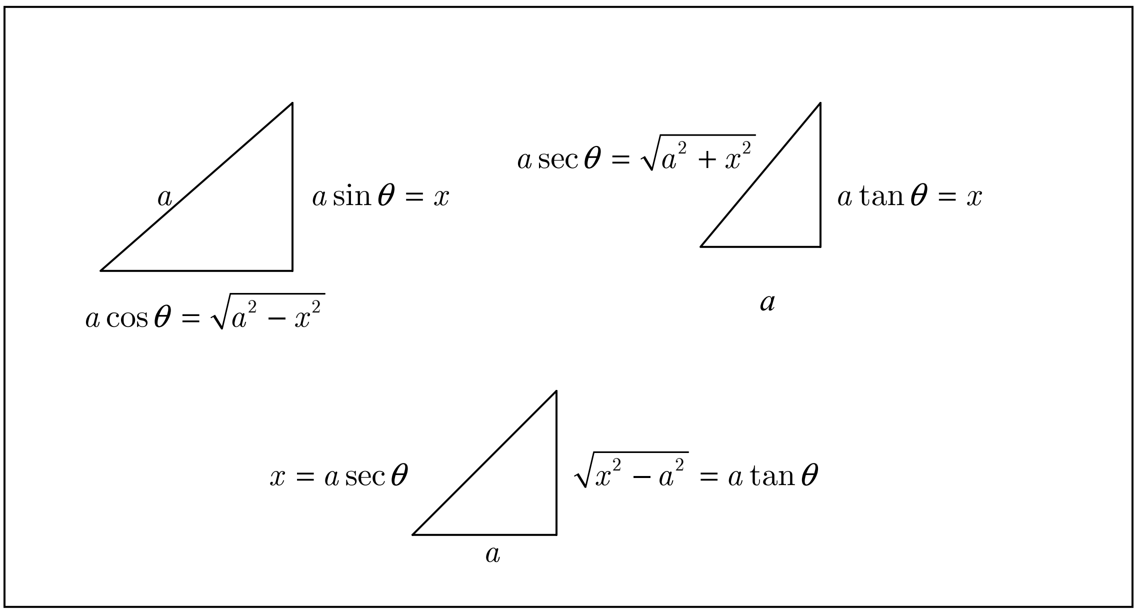
a) $\int \sin^2 x dx$

b) $\int_0^{2\pi} \sin^4 x \cos^2 x dx$

Trigonometric Substitutions

It is possible to simplify certain integrals containing radicals by replacing x with a trigonometric function.

If the integrand contains this	Try substituting this
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$



Example 3: Using Trigonometric Substitution

Evaluate $\int \frac{dx}{x^2\sqrt{4 - x^2}}$

Hint: Use the substitution $x = 2\sin\theta$

Example 4: Using Trigonometric Substitution

$\int \frac{\sqrt{x^2-1}}{x}dx$ Hint: Let $x=\sec\theta$.

Example 5: Using Trigonometric Substitution

$\int \frac{dx}{x^2\sqrt{x^2+4}}$ Hint: Let $x=2\tan\theta$.

Example 6: Using Trigonometric Substitution

$\int \frac{x^2 dx}{\sqrt{a^2 - x^2}}$ Hint: Let $x = a\sin\theta$.

Integration Using Partial Fraction Decomposition

In this section, we examine what to do in the face of a rational function that can't be integrated using substitution. Recall that every polynomial can be factored into a product of linear factors with real zeros and quadratic factors with complex zeros.

Case 1: Denominator is a product of distinct linear factors:

$$\frac{P(x)}{Q(x)}=\frac{P(x)}{(ax+b)(cx+d)}=\frac{A}{ax+b}+\frac{B}{cx+d}$$

Case 2: Denominator is a product of linear factors and some are repeated:

$$\frac{P(x)}{Q(x)}=\frac{P(x)}{(ax+b)(cx+d)^2}=\frac{A}{ax+b}+\frac{B}{cx+d}+\frac{C}{(cx+d)^2}$$

Case 3: Denominator contains irreducible quadratic factors, none of which is repeated:

$$\frac{P(x)}{Q(x)}=\frac{P(x)}{(ax+b)(cx^2+dx+e)}=\frac{A}{ax+b}+\frac{Bx+C}{cx^2+dx+e}$$

Example 1: Integration Using Partial Fraction Decomposition

$$\int \frac{x^3+x}{x-1}dx$$

Hint: First use polynomial division

Some rational integrands have denominators that can be factored into products of linear factors.

Example 2: Integration Using Partial Fraction Decomposition

Factor the denominator into a product of distinct linear factors.

$$\int \frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x} dx$$

Example 3: Integration Using Partial Fraction Decomposition

Factor the denominator into a product of linear factors that are not distinct.

$$\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx$$

Example 4: Integration Using Partial Fraction Decomposition

Factor the denominator into a product of linear and quadratic factors that are irreducible over the reals.

$$\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$$