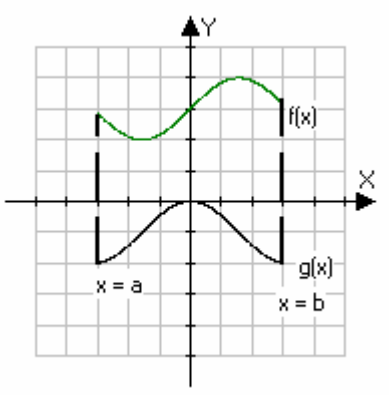
Section 6 The Area between Two Curves

Suppose a region R is enclosed by the graphs of the two functions $f(x)$ and $g(x)$ and the vertical lines $x = a$ and $x = b$. Assuming $f(x) \geq g(x)$ for all *x* in the interval $a \leq x \leq b$, then the area between the curves is given by

Area = $\int_a^b (f(x) - g(x))dx$



Example 1: **Finding the Area between Two Curves**

Find the area of the region bounded by the graphs $f(x) = \sec^2 x$ and $g(x) = \sin x$ from 0 to $\frac{\pi}{4}$.

Example 2: Finding the Area between Two Curves

Find the area of the region bounded by the graphs $f(x) = 2 - x^2$ and $g(x) = -x$.

Example 3: Finding the Area between Two Curves

Find the area of the region bounded by the graphs $f(x) = 4 - x^2$ and $g(x) = -x + 2$ from $x = -2$ to $x = 3$.

Boundaries with Changing Formulas

Example 4: Finding the Area between Two Curves

Find the area of the region in the first quadrant that is bounded above by the graph of $f(x) = \sqrt{x}$ and below by the *x*-axis and the line $g(x) = x - 2$.

Integration with Respect to *y*

Area = $\int_c^d (f(y) - g(y))dy$

Example 5: Finding the Area between Two Curves

Find the area of the region in the first quadrant that is bounded above by the graph of $f(x) = \sqrt{x}$ and below by the *x*-axis and the line $g(x) = x - 2$ using integration with respect to *y*.

Example 6: Finding the Area between Two Curves

Find the area of the region in the first quadrant bounded by the graphs of $f(x) = x - 1$ and $y^2 = 2x + 6$ in two ways.