Name _____

Test 5
Calculus 2
Mike Huff
Spring 2009

Name_____ ***Test 3 Calculus 2 Mike Huff Spring 2009***

(8 points)

1. A function $y(x)$ satisfies the differential equation $\frac{dy}{dx} = y^2 - 4$.

   a) What are the constant (equilibrium) solutions of the equation?

   b) For what values of y is y increasing?

   c) For what values of y is y decreasing?

(6 points)

2. Solve the differential equation $y' = xy$ subject to the initial condition $y(0) = 3$. From your solution, find the value of $y(1)$.

(8 points)

3. Solve the differential equation $y' = -y$ subject to the initial condition $y(0) = \frac{1}{2}$. From your solution, find the value of $y(-2)$.

(8 points)

4. Solve the differential equation: $\frac{dy}{dx} = A - By$.

(8 points)

5. Consider the differential equation $\frac{dx}{dt} = \frac{2tx + x}{t^2 + t}$.

   a) Find the general solution of the differential equation.

   b) Find the solution that satisfies the initial condition $x(1) = 1$.

(8 points)

6. Find the solution of the initial value problem $y' = \frac{y}{x}$, $y(1) = 2$.

(6 points)

7. Determine the solution of the differential equation $\frac{dy}{dt} = y$ where $y(0) = 1$

   a) Use the solution $y(t)$ to calculate $y(1)$

   b) Use Euler's method with step-size $h = 0.2$ to estimate $y(1)$ by filling in the following table for the equation. Find the error in each of your estimates.

x	*y*	*Error*
0		*0*
0.2		
0.4		
0.6		
0.8		
1		

(8 points)

8. Newton's Law of cooling is described by the differential equation $\frac{dT}{dt} = k(T - T_a)$.

   a) Find the solution of this differential equation.

   b) Solve the IVP $T(0) = 300$.

   c) When a cake is removed from the oven its temperature is 300° F. Three minutes later it has cooled to 200° F. How long will it take to cool to 75° F?

(8 points)

9. In a model of epidemics, let $y(t)$, in thousands, be the number of infected individuals in the population at time t, in days. If we assume that the infection spreads to all those who are susceptible, one possible solution for $y(t)$ is given by the solution of $\frac{dy}{dt} = ky(P - y)$ where k is a positive constant which measures the rate of infection and P, in thousands, is the total population in this situation. Find the solution of this differential equation if $y(0) = 2$.

(6 points)

10. A model of the seasonal changes in daylight hours x is given by

$$\frac{dx}{dt} = kx\cos(\omega t),$$

where k and ω are constants.

a) Find the general solution of the differential equation.

(8 points)

11. A tank contains 200g of brine with 5 lb of dissolved salt. Brine containing 0.1 pounds of salt per gallon enters the tank at a rate of 2 g/min. The solution is kept thoroughly mixed and drains from the tank at the same rate.

    a) Find the amount of salt in the tank after t minutes?

    b) How much salt is in the tank after 20 minutes?

    c) As the time increases without bound, what happens to the amount of salt in the tank? What is the concentration then?

(4 points)

12. A bacteria culture starts with 200 bacteria and triples in size every half hour. After 2 hours, how many bacteria are there?

(4 points)

13. The radioactive isotope Bismuth-210 has a half of 5 days. How many days does it take for 87.5% of a given amount to decay?

(4 points)

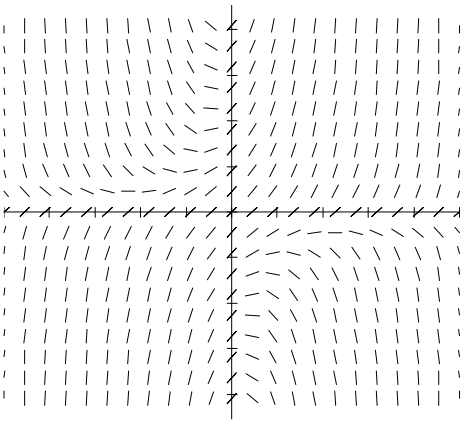
14. A bacteria population grows at a rate proportional to its size. The count was 400 after 2 hours and 25,000 after 6 hours. In how many minutes does the population double?

(6 points)

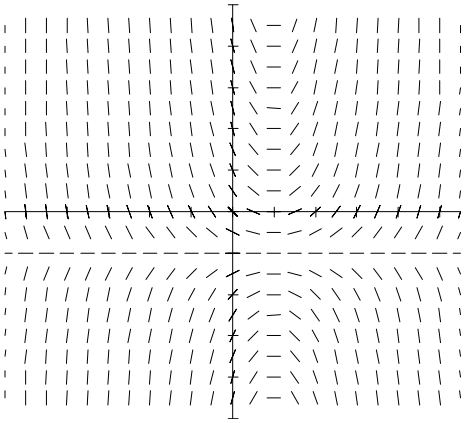
15. Identify the direction field for the given differential equation and then make a sketch of the solution with the given initial value on the correct direction field.

$$y' = (x-1)(y+1), \qquad y(0) = 2$$

a.



b.



c.