Show all work on the test paper for partial credit. <u>No answers accepted without supporting work</u>. It helps to label both the method you are using: u-sub, integration by parts, etc. and the appropriate parts: u, du, v etc.

(8 points)

1. Do both a) and b):

a) $\int_0^{\pi/2} \frac{\sin x}{(3+2\cos x)^2}\,dx$ $= -\frac{1}{2}\int_5^3 u^{-2}\,du$

Let $u = 3 + 2\cos x$

then $du = -2\sin x\,dx$.

$= \frac{1}{2}\int_3^5 u^{-2}\,du$

$= -\frac{1}{2}u^{-1}\Big|_3^5$

$= -\frac{1}{2}\left[\frac{1}{5} - \frac{1}{3}\right] = \frac{1}{15}$

b) $\int x\sec(x^2-5)\,dx$ $= \frac{1}{2}\int \sec u\,du$

Let $u = x^2 - 5$

then $du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\ln|\sec u + \tan u| + C$

$= \frac{1}{2}\ln|\sec(x^2-5) + \tan(x^2-5)| + C$

(7 points)

2. A particle moves along a straight line with a velocity $v(t) = te^{-t}$ meters per second after t seconds. How far will it travel during the first 12 seconds?

$$d(12) = \int_0^{12} te^{-t}\,dt = -te^{-t}\Big|_0^{12} + \int_0^{12} e^{-t}\,dt$$

IBP: $u = t$ $dv = e^{-t}dt$

$du = dt$ $v = -e^{-t}$

$$= -te^{-t} - e^{-t}\Big|_0^{12}$$

$$= -12e^{-12} - e^{-12} + 1$$

$$\boxed{= 1 - 13e^{-12}\,m \approx 0.99992\,m}$$

(7 points)

3. $\int \frac{x^3}{\sqrt{x^4+9}}\,dx$ $= \frac{1}{4}\int u^{-1/2}\,du$

Let $u = x^4+9$

then $du = 4x^3dx$

$\frac{1}{4}du = x^3dx$

$$= \frac{1}{4}\left[2u^{1/2}\right] + C$$

$$\boxed{= \frac{1}{2}\sqrt{x^4+9} + C}$$

(7 points)

4. $\int \tan^4 x dx = \int \tan^2 x \tan^2 x \, dx$

$= \int (\sec^2 x - 1)\tan^2 x \, dx$

$= \int \sec^2 x \tan^2 x \, dx - \int \tan^2 x \, dx$

$\boxed{= \frac{1}{3}\tan^3 x - \tan x + x + C}$

$\int \tan^2 x \, dx$
$= \int (\sec^2 x - 1) dx$
$= \tan x - x$

$\int \sec^2 x \tan^2 x \, dx = \tan^3 x - 2\int \sec^2 x \tan^2 x \, dx$

IBP: $u = \tan^2 x$ $\quad dv = \sec^2 x$

$du = 2\tan x \sec^2 x \, dx$ $\quad v = \tan x$

$\Rightarrow 3\int \sec^2 x \tan^2 x \, dx = \tan^3 x \Rightarrow \int \sec^2 x \tan^2 x \, dx = \frac{1}{3}\tan^3 x$

(7 points)

5. $\int_0^1 \frac{e^x}{1+e^{2x}} dx = \int_1^e \frac{1}{1+u^2} du = \tan^{-1}(u)\Big|_1^e$

Let $u = e^x$

then $du = e^x dx$

$(e^x)^2 = e^{2x}$

$= \tan^{-1}(e) - \tan^{-1}(1)$

$\boxed{= \tan^{-1}(e) - \frac{\pi}{4}}$

≈ 0.43288

(7 points)

6. $\int \frac{1}{\sqrt{49-x^2}}dx$ $= \int \frac{7\cos\theta \, d\theta}{\sqrt{49-49\sin^2\theta}}$

$x = 7\sin\theta$

$dx = 7\cos\theta \, d\theta$

$= \int \frac{\cos\theta}{\sqrt{\cos^2\theta}} d\theta$

$= \int d\theta$

$= \theta$

$\boxed{= \sin^{-1}\left(\frac{x}{7}\right) + C}$

(7 points)

7. $\int \frac{x}{\sqrt{49-x^2}}dx$ $= -\frac{1}{2}\int \frac{1}{\sqrt{u}} du$

Let $u = 49 - x^2$

then $du = -2x\,dx$

$-\frac{1}{2} du = x\,dx$

$= -\frac{1}{2}\int u^{-1/2} du$

$= -u^{1/2} + C$

$\boxed{= -\sqrt{49-x^2} + C}$

(7 points)

8. $\int_0^{\frac{\pi}{6}} e^x \sin x dx$

$\int e^x \sin x\, dx = e^x \sin x - \int e^x \cos x\, dx$

IBP: $u = \sin x \quad dv = e^x dx$

$du = \cos x\, dx \quad v = e^x$

IBP again $\quad u = \cos x \quad dv = e^x dx$

$du = -\sin x\, dx \quad v = e^x$

$= e^x \sin x - e^x \cos x - \int e^x \sin x\, dx$

$\therefore \int_a^{\pi/6} e^x \sin x\, dx = \frac{1}{2} e^x (\sin x - \cos x) + C$

$\int_0^{\pi/6} e^x \sin x = \frac{1}{2} e^{\pi/6} (\sin \frac{\pi}{6} - \cos \frac{\pi}{6}) - \frac{1}{2}(\sin 0 - \cos 0)$

$= \frac{1}{2} e^{\pi/6} (\frac{1}{2} - \frac{\sqrt{3}}{2}) + \frac{1}{2}$

(7 points)

9. $\int 4x \sec^2 2x dx$

Int by parts:

Let $u = 4x \quad dv = \sec^2 2x$

$du = 4\,dx \quad v = \frac{1}{2} \tan 2x$

$= 2x \tan 2x + \int -2 \tan 2x\, dx$

Let $u = \cos 2x$

then $du = -2 \sin 2x\, dx$

$= 2x \cdot \tan 2x + \int \frac{-2 \sin 2x}{\cos 2x} dx$

$= 2x \cdot \tan 2x + \int \frac{du}{u}$

$= 2x \tan 2x + \ln|\cos 2x| + C$

I find the indefinite integral first:

(7 points)

10. $\int_{-\infty}^{\infty} 2xe^{-x^2}\,dx$

$\int 2xe^{-x^2}dx$ u-sub w/ $u=-x^2$ then $du=-2xdx$, $-du=2xdx$

$= -\int e^u du = -e^{-x^2}+C$

$\int_{-\infty}^{\infty} f(x)dx = \lim_{b\to\infty}\int_0^b f(x)dx + \lim_{a\to-\infty}\int_a^0 f(x)dx$

$= \lim_{b\to\infty}\left[-e^{-b^2}+1\right] + \lim_{a\to-\infty}\left[-1+e^{-a^2}\right]$

$= 1 - 1$

$= 0$

(7 points)

11. $\int \frac{x+4}{x^3+3x^2-10x}\,dx$

$\frac{x+4}{x(x^2+3x-10)} = \frac{x+4}{x(x+5)(x-2)} = \frac{A}{x} + \frac{B}{x+5} + \frac{C}{x-2}$

$= \int\left(-\frac{2}{5}\cdot\frac{1}{x} - \frac{1}{35}\,\frac{1}{x+5} + \frac{3}{7}\cdot\frac{1}{x-2}\right)dx$

$= -\frac{2}{5}\ln|x| - \frac{1}{35}\ln|x+5| + \frac{3}{7}\ln|x-2| + C$ ✓

(7 points)

12. $\int \cos^5 t\,dt$ $= \int \cos^4 t \cos t\, dt$

$= \int (1-\sin^2 t)^2 \cos t\, dt$ Use u-substitution with $u = \sin t$ then $du = \cos t\, dt$

$= \int (1-u^2)^2\, du$

$= \int (1 - 2u^2 + u^4)\, du$

$= u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C$

$= \sin t - \frac{2}{3}\sin^3 t + \frac{1}{5}\sin^5 t + C$

(7 points)

13. Find $g'(x)$ given $g(x) = \int_0^{\cos x} \frac{1}{\left(1+t^2\right)^2} dt$.

$g'(x) = \frac{1}{(1+\cos^2 x)^2} \cdot (-\sin x)$

(7 points)

14. $\int_0^3 \frac{1}{x-1} dx$.

$$= \overset{(1)}{\int_0^1} \frac{1}{x-1}\, dx + \overset{(2)}{\int_1^3} \frac{1}{x-1}\, dx$$

Consider (1):

$$\int_0^1 \frac{1}{x-1}\, dx = \lim_{b \to 1^-} \int_0^b \frac{1}{x-1}\, dx$$

$$= \lim_{b \to 1^-} \Big[\ln|x-1| \Big]_0^b$$

$$= \lim_{b \to 1^-} \ln|b-1|$$

$$= -\infty \text{ (diverses)}$$

Since (1) diverses, the original integral diverses.