

1. $\int \frac{1}{\sqrt{1+e^x}} dx$

7. $\int \frac{x+1}{x^2-2x+10} dx$

2. $\int \sec^3 x dx$

8. $\int \sin(\ln x) dx$

3. $\int_0^{2\pi} \sin mx \sin nx dx$, where m and n are integers with $m \neq \pm n$.

9. $\int_0^{\pi/4} \sqrt{1+\cos 4x} dx$

4. $\int_0^{1/\sqrt{2}} 2x \sin^{-1}(x^2) dx$

10. $\int \frac{1}{x^2 \sqrt{16-x^2}} dx$

5. $\int x^3 \sqrt{x+1} dx$

11. $\int_0^{\infty} x^4 e^{-x} dx$

6. $\int \frac{1}{1+\tan^2(x)} dx$

12. $\int_0^2 \frac{dx}{1-x^2}$

13. **The Gamma Function:** Euler's Gamma function is defined by the formula:

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt, \quad x > 0.$$

- Show that $\Gamma(1) = 1$.
- Apply integration by parts to the integral for $\Gamma(x+1)$ to show that $\Gamma(x+1) = x\Gamma(x)$.

Do two of the following three problems:

14. Use (a) Simpson's Rule (b) the Trapezoidal Rule, and (c) the Midpoint Rule with $n = 6$ to approximate π using the equation

$$\pi = \int_0^1 \frac{4}{1+x^2} dx.$$

Find the maximum possible error using each approach.

15. A single infected individual enters a community of n susceptible individuals. Let x be the number of newly infected individuals at time t . The common **epidemic model** assumes that the disease spreads at a rate proportional to the product of the total number infected and the number not yet infected. Thus

$$\frac{dx}{dt} = k(x+1)(n-x)$$

and we obtain

$$\int \frac{1}{(x+1)(n-x)} dx = \int k dt$$

Use partial fraction decomposition to integrate the left-hand side and then solve for x as a function of t .

16. Use the substitution $u = \tan\left(\frac{\theta}{2}\right)$ and the identity $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ to evaluate the integral

$$\int_0^{\pi/2} \frac{d\theta}{4 \sin \theta + 3 \cos \theta}$$

Calculus 2

Take-Home Test 1

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Directions: Copy all finished work neatly onto paper. Put the finished problems *in order* and staple the work together.

All work must be done neatly with detailed explanations. That includes showing what u is when making substitutions, and showing u , du , v , and dv for any integration by parts. Write out each term for numerical techniques, etc. You may check your work using calculators or computers but you must show each step of the calculation for full credit.

You may share ideas but you may not copy another student's work. You may get help from the lab on the

techniques but they may not work out the problem for you.