

Name _____ Test 1 Summer 2009 Calculus 2 Mike Huff

There are 17 problems to do. Each problem is worth 5 points for a total of 85 points.

1. $\int (x^{2/3} + \frac{x-4}{\sqrt{x^2-8x+1}}) dx$

2. $\int \tan^3 3x \sec^4 3x dx$

3. $\int \sin 2x \cos 2x dx$

4. $\int \tan^{-1} \left(\frac{x}{2} \right) dx$

5. $\int \frac{x^2}{\sqrt{2+3x}} dx$

6. $\int (\ln x)^2 dx$

7. $\int \frac{dx}{x^2+2x+5}$

8. $\int (\cot x) [\ln(\sin x)] dx$

9. $\int \frac{5x^2+20x+6}{x(x+1)^2} dx$

10. $\int_0^3 \frac{x^3}{\sqrt{x^2+9}} dx$

11. $\int \frac{\sqrt{4x^2+9}}{x^4} dx$

12. $\int_1^\infty (1-x)e^{-x} dx$

13. $\int_0^{\pi/3} \frac{\sin^3 x dx}{\sqrt{\cos x}}$

14. The work done against gravity in propelling an object with mass m kg to an altitude of h m above the surface of the earth is given by

$$W = \int_{6.37 \times 10^6}^{6.37 \times 10^6 + h} \frac{GM_E m}{r^2} dr$$

where 6.37×10^6 m is the radius of the earth, $G \approx 6.667 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$, and $M_E \approx 5.90 \times 10^{24}$ kg is the mass of the earth.

- a) Find the work required to launch a 1000-kg satellite vertically to an altitude of 1000 km. The formula shows that the work is dependent on h .
- b) Show that $\int_{6.37 \times 10^6}^\infty \frac{GM_E m}{r^2} dr$ is convergent.

15. A string stretched between two points $(0,0)$ and $(0,2)$ is plucked by displacing the string h units at its midpoint. The motion of the string is modeled by the Fourier Sine Series whose coefficients are given by

$$b_n = h \int_0^1 x \sin\left(\frac{n\pi x}{2}\right) dx + h \int_1^2 (-x+2) \sin\left(\frac{n\pi x}{2}\right) dx,$$

where n is an integer. Evaluate b_n .

Take home test – no help from the Learning Lab.

16. Use (a) Simpson's Rule (b) the Trapezoidal Rule, and (c) the Midpoint Rule with $n = 6$ to approximate $\frac{1}{\sqrt{2\pi}} \int_{-3}^3 e^{-x^2/2} dx$.

Find the maximum possible error using each approach.

Do one of the following two problems.

17. In a chemical reaction, one unit of compound Y and one unit of compound Z are converted into a single unit of compound X . If the amount of compound X formed and the rate of formation of X is proportional to the amount of unconverted compounds Y and Z , then

$$\frac{dx}{dt} = k(y_0 - x)(z_0 - x)$$

where y_0 and z_0 are the initial amounts of substances Y and Z , respectively. From the above equations we obtain,

$$\int \frac{1}{(y_0 - x)(z_0 - x)} dx = \int k dt$$

Use partial fraction decomposition to integrate the left-hand side and then solve for x in terms of t .

18. Use the substitution $u = \tan\left(\frac{\theta}{2}\right)$ and the identity $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ to evaluate the integral

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Calculus 2
Test 1-Part 1
Take-Home
Mike Huff
Summer 2009

Directions: Copy all finished work neatly onto paper. Put the finished problems ***in order*** and staple the work together. Typed is even better! All work must be done neatly with detailed explanations. That includes showing what u is when making substitutions, and showing u , du , v , and dv for any integration by parts. Write out each term for numerical techniques, etc. You may check your work using calculators or computers but you must show each step of the calculation for full credit. You may share ideas but *you may not copy another student's work.* *You may not get help from any Learning Lab.*

Deadline _____