

Name _____ Calculus 2 Summer 2010 Final Exam Huff

Part 1 Using Known Series

1. Express $\frac{x}{1-x^2}$ as a power series.

2. Express $\int_0^{0.3} \frac{1}{1+x^3} dx$ as a power series.

3. Express $\int \cos x^2 dx$ as a Maclaurin series.

4. Find Maclaurin series for the functions e^{4x} , $e^{-x^2/2}$, and e^{-x} .

Part 2 Finding Taylor Series from Scratch

5. Find the 4th-degree Taylor polynomial centered at $x = 4$ for the function $f(x) = \sqrt{x}$. Use the polynomial to approximate $\sqrt{4.1}$.

6. Find the Taylor series at $a = \frac{\pi}{2}$ for the function defined by $f(x) = \cos x$ and determine its interval of convergence.

7. Find the Maclaurin series for the function defined by $f(x) = \sin 2x$ and find its interval of convergence. **(Show all the work here.)**

Part 3 Differential Equations

8. A function $y(x)$ satisfies the differential equation $\frac{dy}{dx} = 9 - y^2$.
- a) What are the constant (equilibrium) solutions of the equation? Is each stable or unstable?

 - b) For what values of y is y increasing?

 - c) For what values of y is y decreasing?

 - d) Classify all equilibria as stable or unstable.
9. Solve the differential equation $y' = -2y$ subject to the initial condition $y(0) = 6$. From your solution, find the value of $y(-1)$.

10. Solve the differential equation $\frac{dy}{dt} = 3y + 1$ subject to the initial condition $y(0) = 2$. From your solution, find the value of $y(2)$.

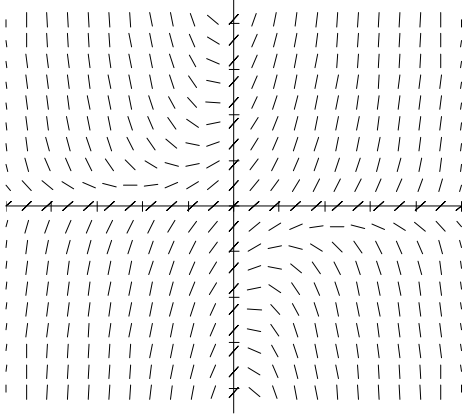
11. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y^2}{x}$.

12. Solve the differential equation: $\frac{dy}{dx} = y(A - By)$.

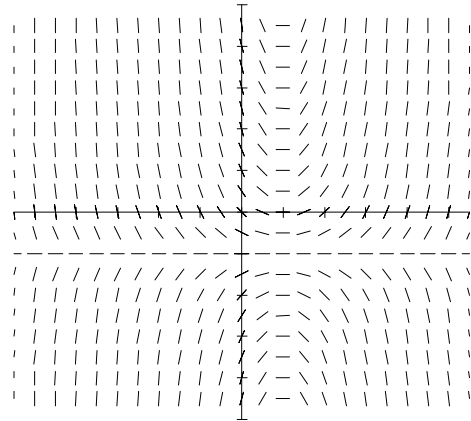
13. Identify the direction field for the given differential equation and then make a sketch of the solution with the given initial value on the correct direction field.

$$y' = (x - 1)(y + 1), \quad y(0) = 2$$

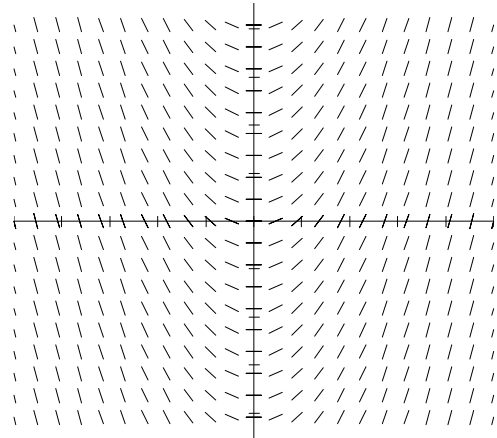
a.



b.



c.



14. Newton's Law of cooling is described by the differential equation

$\frac{dT}{dt} = k(T - T_a)$. Assume the room temperature is 73°F . If it takes 15 minutes for a can of soda to warm from 35°F to 38°F ., find out how long will it takes to warm a can of soda from 35°F to 45°F

15. A tank contains 500 gallons of brine with 15 lb of dissolved salt. Brine containing 0.5 pounds of salt per gallon enters the tank at a rate of 5 gallons per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate.
- a) Find the amount of salt in the tank after t minutes?
 - b) How much salt is in the tank after 30 minutes?
 - c) As the time increases without bound, what happens to the amount of salt in the tank?
 - d) What is the concentration then?