Mixing Problems

Consider a tank containing a solution – a mixture that contains both a solvent and a solute. At time *t* = 0, the tank contains an amount of solute $x(0) = x_0$. Suppose also that a solution with concentration c_i enters the tank at a constant rate of r_i and that a thoroughly mixed solution flows out at a constant rate of r_o. The rate of flow of solute out of the tank depends on the concentration $c_o(t) = x(t)/V(t)$ of the solute in the solution at time *t*.

$$\Delta x = \text{amount in} - \text{amount out} \approx r_i c_i \Delta t - r_o c_o \Delta t$$

$$\frac{\Delta x}{\Delta t} \approx r_i c_i - r_o c_o$$

Finally, if we take the limit as $\Delta t \to 0$, we obtain the differential equation

$$\frac{dx}{dt} = r_i c_i - r_o c_o = r_i c_i - \frac{r_o}{V} x$$

Example 1: Mixing problem

At time $t = 0$, a 100 gallon tank contains 50 *lbs* of salt. Assume that water containing $\frac{1}{4} lb/gal$ is entering the tank at rate of $3 gal/\min$ and that the well-stirred mixture is draining from the tank at the same rate. Find $x(t)$ the amount of salt in the tank at any time.

$$\frac{dx}{dt} = r_i c_i - \frac{r_o}{V} x$$

$$\frac{dx}{dt} = 3\frac{gal}{\min}\frac{(1/4)lb}{gal} - \frac{1}{100gal} 3\frac{gal}{\min} x(t) lb$$

$$\frac{dx}{dt} = \frac{3}{4} - \frac{3x}{100}$$

Example 2: Mixing problem

A tank with a capacity of 500 gallons originally contains 200 g of water with 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flow out of the tank at a rate of 2 gal/min. Find the amount of salt in the tank at any time prior to the instant when the tank begins to overflow. Find the concentration (in pounds per gallon) of salt in the tank when it is on the point of overflowing. Compare this concentration with the theoretical limiting concentration if the tank had infinite capacity.