Area = 49π

Area of ½ slice = $\dfrac{49\pi}{6}$

Equation of top half of circle = $\sqrt{49 - x^2}$

We must find c so that the area of ½ slice is $\int_c^7 \sqrt{49 - x^2}\,dx = \dfrac{49\pi}{6}$.

$\dfrac{49\pi}{6} = \int_c^7 \sqrt{49 - x^2}\,dx$ Use trigonometric substitution and let $x = 7\sin(\theta)$ so that $dx = 7\cos(\theta)d\theta$. Note that, when $x = c$ we have $\theta = \sin^{-1}\left(\dfrac{c}{7}\right)$ and when $x = 7$ we have $\theta = \sin^{-1}\left(\dfrac{7}{7}\right) = \dfrac{\pi}{2}$.

Using this substitution, we have

$$\frac{49\pi}{6} = \int_{\sin^{-1}\left(\frac{c}{7}\right)}^{\pi/2} \sqrt{49 - 49\sin^2\theta}\,7\cos\theta d\theta = \int_{\sin^{-1}\left(\frac{c}{7}\right)}^{\pi/2} 49\sqrt{1 - \sin^2\theta}\cos\theta d\theta$$

$$= 49\int_{\sin^{-1}\left(\frac{c}{7}\right)}^{\pi/2} \sqrt{\cos^2\theta}\cos\theta d\theta$$

$$= 49\int_{\sin^{-1}\left(\frac{c}{7}\right)}^{\pi/2} \cos^2\theta d\theta$$

$$= 49\int_{\sin^{-1}\left(\frac{c}{7}\right)}^{\pi/2} \left[\frac{1}{2} + \frac{1}{2}\cos 2\theta\right] d\theta$$

$$= 49\left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_{\sin^{-1}\left(\frac{c}{7}\right)}^{\pi/2}$$

$$= 49\left[\frac{\pi}{4} + \frac{1}{4}\sin\pi - \frac{1}{2}\sin^{-1}\left(\frac{c}{7}\right) - \frac{1}{4}\sin\left(2\sin^{-1}\left(\frac{c}{7}\right)\right)\right]$$

$$\frac{49\pi}{6} = 49\left[\frac{\pi}{4} - \frac{1}{2}\sin^{-1}\left(\frac{c}{7}\right) - \frac{1}{4}\sin\left(2\sin^{-1}\left(\frac{c}{7}\right)\right)\right]$$

$$\frac{\pi}{6} = \frac{\pi}{4} - \frac{1}{2}\sin^{-1}\left(\frac{c}{7}\right) - \frac{1}{4}\sin\left(2\sin^{-1}\left(\frac{c}{7}\right)\right)$$

Solving for c graphically, we have $c \approx 1.8545246$ inches.