

The rocket will have height $H = \int_0^{60} v(t)dt$ after 60 seconds.

$$\begin{aligned} H &= \int_0^{60} \left[-gt - v_e \ln \left(\frac{m - rt}{m} \right) \right] dt \\ &= -g \left(\frac{1}{2} t^2 \right) \Big|_0^{60} - v_e \left[\int_0^{60} \ln(m - rt) dt - \int_0^{60} \ln m dt \right] \\ &= -1800g + v_e (\ln m)(60) - v_e \int_0^{60} \ln(m - rt) dt \end{aligned}$$

The last integral can be done using integration by parts. Let $u = \ln(m - rt)$ and $dv = dt$, then $du = \frac{-r}{m - rt} dt$, and $v = t$. Then the integral $\int_0^{60} \ln(m - rt) dt$ becomes:

$$\begin{aligned} \int_0^{60} \ln(m - rt) dt &= t \ln(m - rt) \Big|_0^{60} + \int_0^{60} \frac{rt}{m - rt} dt \\ &= 60 \ln(m - 60r) + \int_0^{60} \left[-1 + \frac{m}{m - rt} \right] dt \\ &= 60 \ln(m - 60r) + \left[-t - \frac{m}{r} \ln(m - rt) \right]_0^{60} \\ &= 60 \ln(m - 60r) + \left[-60 - \frac{m}{r} \ln(m - 60r) \right] + \frac{m}{r} \ln m \end{aligned}$$

It follows that

$$\begin{aligned} H &= -1800g + 60v_e \ln m - v_e \left[60 \ln(m - 60r) + \left(-60 - \frac{m}{r} \ln(m - 60r) \right) + \frac{m}{r} \ln m \right] \\ &= -1800g + 60v_e \ln m - 60v_e \ln(m - 60r) + 60v_e + \frac{m}{r} v_e \ln(m - 60r) - \frac{m}{r} v_e \ln m \end{aligned}$$

Substituting $g = 9.8$, $m = 30000$, $r = 160$, and $v_e = 3000$, we have

$$\begin{aligned} H &= -1800(9.8) + 60 \cdot 3000 \ln 30000 - 60 \cdot 3000 \ln(30000 - 60 \cdot 160) \\ &\quad + 60 \cdot 3000 + \frac{30000}{160} \cdot 3000 \ln(30000 - 60 \cdot 160) - \frac{30000}{160} 3000 \ln 30000 \approx 14,844m. \end{aligned}$$