Section 6 Power Series

Definition: A series of the form $\sum_{n=0}^{\infty} a_n x^n$ is called a **power series** where x is a variable and the a_n 's are constants called the coefficients of the series. We can also define power series centered at a: $\sum_{n=0}^{\infty} a_n (x-a)^n$

Convergent power series are often used to represent functions.

Example 1: **Representing a function using a power series**

The series $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \frac{1}{1-x}$ as long as $|x| < 1$.

Example 2: Convergence of Power Series

For what values of x does the series $\sum_{n=0}^{\infty} n!x^n$ converge?

Example 3: Convergence of Power Series

For what values of x does the series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$ converge?

Theorem: For a given power series $\sum_{n=0}^{\infty} a_n (x-a)^n$ there are only three possibilities:

a) The series converges only when $x = a$.
b) The series converges for all x.
c) There is a positive number *R* such that the series converges if $|x-a| < R$ and diverges when $|x-a| > R$.

Definitions: The number *R* is called the **radius of convergence** and the interval of *x* values for which the series converges is called the **interval of convergence**.

Example 4: Finding the interval and radius of convergence of power series

Find the radius of convergence and the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$

Example 5: Find the interval and radius of convergence of power series

Find the radius of convergence and the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{3^n x^n}{(n+1)^2}$

Example 6: Find the interval and radius of convergence of power series

Find the radius of convergence and the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{4^n \ln n}$