

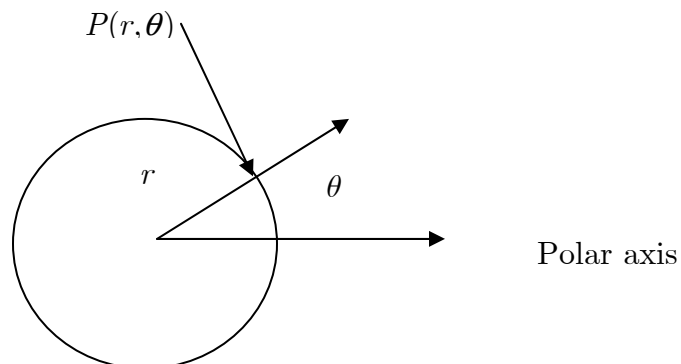
Section 2 Polar Coordinates: Curves in Polar Coordinates

Curves in Polar Coordinates

Points in the plane \mathbb{R}^2 can also be described using a coordinate system different than the Cartesian coordinate system. This system is called the polar coordinate system and was developed by Isaac Newton. In this system, a point in the plane is located using an angle and a distance.

Definition: Polar Coordinate System Select a point in the plane and a ray emanating from this point. The point is called the **pole** and the ray is called the **polar axis**. We measure positive angles θ in a counterclockwise direction from the polar axis. Let r be a number that will represent a distance from the pole. To plot the point P corresponding to the pair of numbers r and θ , proceed as follows:

1. If r is positive, P is the intersection of the circle of radius r whose center is at the pole and the ray of angle θ emanating from the pole.
2. If r is zero, P is the pole no matter what θ is.
3. If r is negative, P is at a distance $|r|$ from the pole on the ray directly opposite the ray of angle θ .



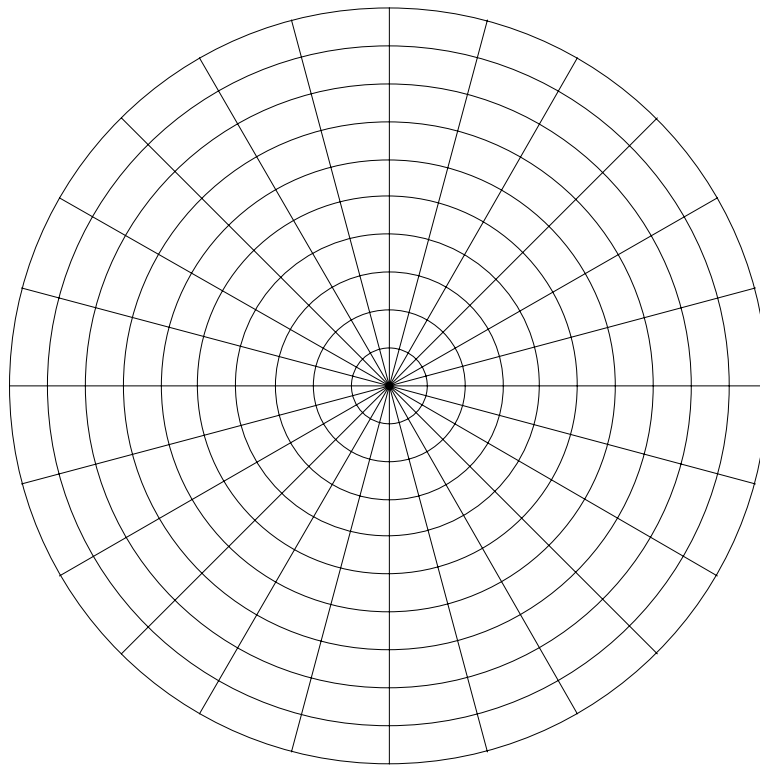
Example 1 Plotting points using polar coordinates

Plot the points whose coordinates are given.

a) $(1, \frac{\pi}{2})$

b) $(5, \pi)$

c) $(-2, \frac{3\pi}{4})$



Converting Polar and Rectangular Coordinates

To convert from polar to rectangular coordinates, we use the formulas

$$x = r \cos \theta \text{ and } y = r \sin \theta$$

Example 2 Converting from Polar to Rectangular Coordinates

Convert the point $\left(1, \frac{\pi}{6}\right)$ from polar to rectangular coordinates.

To convert from rectangular to polar coordinates, we use the formulas

$$r^2 = x^2 + y^2 \text{ and } \tan \theta = \frac{y}{x}$$

Example 3 Converting from Rectangular to Polar Coordinates

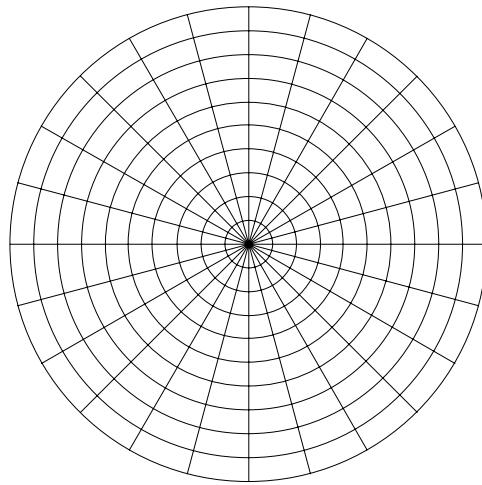
Convert the point $(-1, 1)$ from rectangular to polar coordinates.

Definition: The Graph of a Polar Equation. The graph of a polar equation $r = f(\theta)$ consists of the sets of points that have at least one polar representation (r, θ) whose coordinates satisfy the equation.

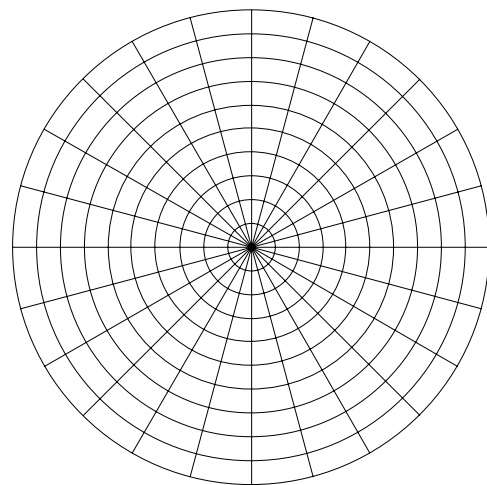
Example 4 Graphing Using Polar Coordinates

Graph the following.

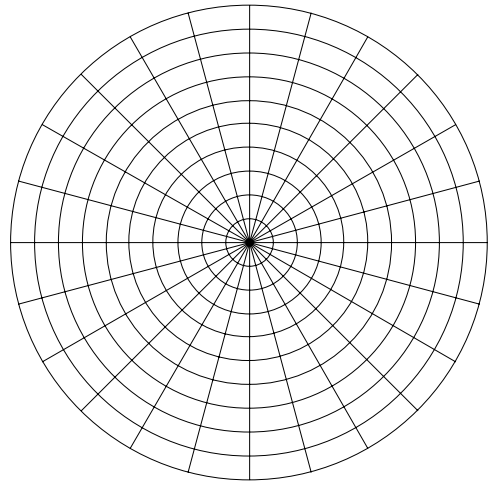
a) $r = 1$



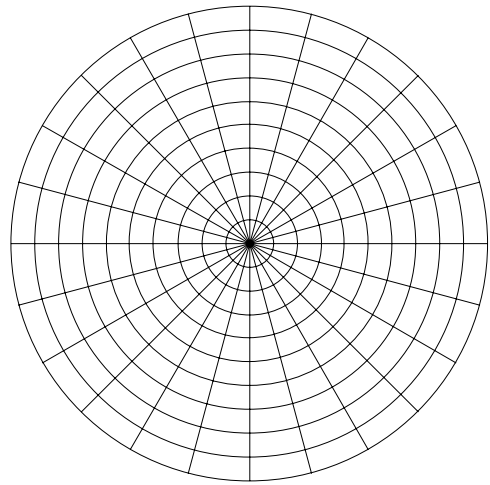
b) $r = 2 \cos \theta$ Find a Cartesian equation for this curve.



c) $r = 1 + \sin \theta$ Cardioid



d) $r = \cos 2\theta$ Four-leaved rose



Tangents to Polar Curves

To find the slope of a curve defined using polar coordinates, we treat θ as a parameter and write the parametric equations of $r = f(\theta)$ as

$$\begin{aligned}x &= r \cos \theta = f(\theta) \cos \theta, \\y &= r \sin \theta = f(\theta) \sin \theta.\end{aligned}$$

We now think of these as parametric equations and use the method for finding tangents to parametric curves that we developed previously.

The derivative $\frac{dy}{dx}$ is then

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$$

Note that the tangent is horizontal whenever $\frac{dy}{d\theta} = 0$ and $\frac{dx}{d\theta} \neq 0$. We also have a vertical tangent whenever $\frac{dx}{d\theta} = 0$ and $\frac{dy}{d\theta} \neq 0$.

Example 5 Finding tangent lines to a polar curve

- a) Find the slope of the tangent line when $\theta = \frac{\pi}{3}$ for the polar curve $r = 1 + \sin \theta$.

- b) Find the points on the graph where the tangent line is vertical or the tangent line is horizontal.