Separable Differential Equations

Definition: A differential equation that can be written in the form $g(y)\frac{dy}{dx} = f(x)$ is called a separable differential equation.

Using differentials we can write this equation in the form $g(y)dy = f(x)dx$ and then integrate both sides to get: $\int g(y)dy = \int f(x)dx$. If $G(y)$ is any antiderivative of $g(y)$ and $F(x)$ is any antiderivative of $f(x)$, then the equation $G(y) = F(x) + C$ defines a family of solutions of the original equation implicitly.

Example 1: Solving a separable differential equation

Solve $\frac{dy}{dx} = -3xy^2$

Example 2: Solving an initial-value problem

Solve $(4y - \cos y)\frac{dy}{dx} - 3x^2 = 0$, $y(0) = 0$

Applications: Growth Models

Unrestricted Growth (Decay) Model

Rate of change is proportional to current population: $\dfrac{dP}{dt} = kP$

Limited Growth Model

Rate of change is proportional to the difference between some maximum population M and the current population: $\frac{dP}{dt} = k(M - P)$

Logistic Growth Model

Rate of change is jointly proportional to the current population and the difference between some maximum population M and the current population:

$$\frac{dP}{dt} = kP(M - P)$$

This models the spread of infectious diseases, the growth of a business, and the spread of a rumor.

Newton's Law of Cooling

Rate at which an object cools is proportional to the difference in temperature between the object and the surrounding atmosphere: $\frac{dT}{dt} = k(T - T_a)$. We use T to represent the temperature of the object at any time t and T_a for the ambient temperature.