Name ____________________ Test 1B Calculus 2 Mike Huff

Show all work on the test paper for partial credit. <u>No answers accepted without supporting work</u>. It helps to label both the method you are using: u-sub, integration by parts, and the appropriate parts: u, du, v, etc.

(8 points)

1. Do both a) and b):

a) $\int (x^{5/2} + \frac{1}{\sqrt{1-x^2}})dx = \frac{2}{7}x^{7/2} + \sin^{-1}x + C$

b) $\int \frac{1}{x(\ln x)^5}dx = \int u^{-5}du = -\frac{1}{4u^4} + C = -\frac{1}{4(\ln x)^4} + C$

$u = \ln x$

$du = \frac{1}{x}dx$

(7 points)

2. A particle moves along a straight line with a velocity $v(t) = t^2e^{-t}$ meters per second after t seconds. How far will it travel during the first 45 seconds?

$\int_0^{45} t^2 e^{-t} dt = -t^2e^{-t} - 2te^{-t} - 2e^{-t} \Big|_0^{45}$

u	dv	±1
t^2	e^{-t}	1
$2t$	$-e^{-t}$	-1
2	e^{-t}	1
0	$-e^{-t}$	-1
		1

$= -\frac{45^2}{e^{45}} - \frac{90}{e^{45}} - \frac{2}{e^{45}} + 0 + 0 + 2$

$= 2 - \frac{2117}{e^{45}}$ meters

≈ 2 m

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(7 points)

3. $\int x\sqrt{x-1}dx$ $= \int (u+1)u^{1/2}du$

Let $u = x-1$ $= \int (u^{3/2} + u^{1/2})du$

then $du = dx$

$= \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$

(7 points)

4. $\int_{-1}^{2} \frac{x}{\sqrt{x+2}}dx$ $= \int_1^4 \frac{u-2}{\sqrt{u}}du$

Let $u = x+2$ $= \int_1^4 (u^{1/2} - 2u^{-1/2})du$

then $du = dx$

limits: $x=-1$ $\Rightarrow u = 1$ $= \frac{2}{3}u^{3/2} - 4u^{1/2}\Big|_1^4$

$x = 2$ $\Rightarrow u = 4$ $= \frac{2}{3}4^{3/2} - 4\sqrt{4} - \left(\frac{2}{3} - 4\right) = \frac{2}{3}$

(7 points)

5. $\int_0^1 \frac{e^x}{1+e^x}dx$ $= \int_2^{e+1} \frac{1}{u}du = \ln|u|\Big|_2^{e+1}$

$= \ln|e+1| - \ln|2|$

Let $u = 1+e^x$

then $du = e^x dx$ $= \ln\frac{e+1}{2}$ $[\approx 0.620]$

limits: $x = 0 \Rightarrow u = 2$

$x = 1 \Rightarrow u = e+1$

(7 points)

6. $\int_{-3}^{3} \frac{x}{\sqrt{1+3x^2}} dx$

The function $f(x) = \frac{x}{\sqrt{1+3x^2}}$ is an odd function, hence the integral evaluates to 0.

(7 points)

7. $\int \cos x \sin(\sin x)\, dx = \int \sin u \, du = -\cos u + C$

$= -\cos(\sin x) + C$

let $u = \sin x$

then $du = \cos x \, dx$

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(7 points)

8. $\int_0^{\frac{\pi}{2}} e^x \sin x dx$

I first find the antiderivative using IBP:

$\int e^x \sin x\, dx = e^x \sin x - \int e^x \cos x\, dx$

$u = \sin x \quad dv = e^x dx \qquad u = \cos x \quad dv = e^x dx$

$du = \cos x\, dx \quad v = e^x \qquad du = -\sin x\, dx \quad v = e^x dx$

$\int e^x \sin x dx = e^x \sin x - e^x \cos x - \int e^x \sin x dx$

$2\int e^x \sin x dx = e^x \sin x - e^x \cos x$

$\therefore \int e^x \sin x dx = \frac{1}{2} e^x \sin x - \frac{1}{2} e^x \cos x + C$

$\int_0^{\pi/2} e^x \sin x dx = \frac{1}{2} e^x \sin x - \frac{1}{2} e^x \cos x \Big|_0^{\pi/2}$

$= \frac{1}{2} e^{\pi/2} \sin\frac{\pi}{2} - \cancel{\frac{1}{2} e^{\pi/2} \cos\frac{\pi}{2}}^{0} - \cancel{\frac{1}{2} e^0 \sin 0}^{0} + \frac{1}{2} e^0 \cos 0 = \frac{1}{2} e^{\pi/2} + \frac{1}{2}$

$[\approx 2.905]$

5

(7 points)

9. $\int_0^1 x \tan^{-1} x dx$

$= \frac{x^2}{2} \tan^{-1} x \Big|_0^1 - \frac{1}{2} \int_0^1 \frac{x^2}{1+x^2} dx$

IBP: $u = \tan^{-1} x \quad dv = x dx$

$du = \frac{1}{1+x^2} dx \quad v = \frac{x^2}{2}$

$= \frac{1}{2} \tan^{-1}(1) - \frac{1}{2} \int_0^1 \left(1 - \frac{1}{1+x^2}\right) dx$

$= \frac{1}{2} \frac{\pi}{4} - \frac{1}{2} \left[x - \tan^{-1}(x)\right]_0^1$

$= \frac{\pi}{8} - \frac{1}{2}\left(1 - \tan^{-1}(1) - (0)\right)$

$= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}$

$= \frac{\pi}{4} - \frac{1}{2}$

$[\approx 0.285]$

(7 points)

10. $\int_{-\infty}^{0} xe^x\,dx$

IBP: $u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$$= \lim_{a\to-\infty} \int_a^0 xe^x\,dx$$

$$= \lim_{a\to-\infty} xe^x\Big|_a^0 - \int_a^0 e^x\,dx \quad \left[= \lim_{a\to-\infty} xe^x - e^x\Big|_a^0\right.$$

$$= \lim_{a\to-\infty} -ae^a - e^x\Big|_a^0$$

$$= \lim_{a\to-\infty} \left(-ae^a - 1 + e^a\right) = \lim_{a\to-\infty} \left(-1 + \frac{1-a}{e^{-a}}\right) \boxed{= -1}$$

We use L'Hopital's $\frac{\infty}{\infty}$ case to get the last limit:

$$\lim_{a\to-\infty} \frac{1-a}{e^{-a}} = \lim_{a\to-\infty} \frac{-1}{-e^{-a}} = 0$$

(7 points)

11. $\int \frac{5x-3}{x^2-2x-3}dx = \int \frac{5x-3}{(x-3)(x+1)}dx = \int \frac{2}{x+1}dx + \int \frac{3}{x-3}dx$

Partial fraction decomposition:

$$\frac{5x-3}{(x-3)(x+1)} = \frac{2}{x+1} + \frac{3}{x-3}$$

$$= 2\ln|x+1| + 3\ln|x-3| + C$$

$$= \ln (x+1)^2|x-3|^3 + C$$

(7 points)

12. $\int \frac{1}{\sqrt{4+x^2}} dx = \int \frac{2\sec^2\theta \, d\theta}{\sqrt{4\tan^2\theta + 4}} = \int \sec\theta \, d\theta$

$= \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$

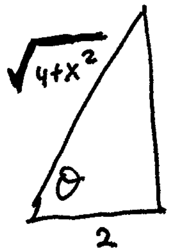
or $\ln\left|\sqrt{4+x^2} + x\right| + C$

use this sub!

$x^2 = 4\tan^2\theta$

$x = 2\tan\theta$

$dx = 2\sec^2\theta \, d\theta$



$\tan\theta = \frac{x}{2}$

$\sec\theta =$

(8 points)

CAH

13. Suppose $\int_{-2}^{2} f(x)dx = 4$, $\int_{2}^{5} f(x)dx = 3$, and $\int_{-2}^{2} g(x)dx = 2$. Decide if the following statements are true or false. Circle the correct choice.

a) $\int_{5}^{2} f(x)dx = -3$ — True or False?

b) $\int_{-2}^{5} (f(x) + g(x))\, dx = 9$ — True or False?

c) $\int_{-2}^{5} f(x)dx = 8$ — True or False?

d) $f(x) \le g(x)$ on the interval $-2 \le x \le 5$ — True or False

(7 points)

14. Find y' given $y = \int_{0}^{\tan x} \frac{dt}{1+t^2}$.

$y' = \frac{1}{1+\tan^2 x} (\tan x)' = \frac{\sec^2 x}{\sec^2 x} = 1$

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(8 points)

15. Compute an approximate value of $\int_0^{\pi/2} x \sin x dx$ using one of the methods developed in class: Midpoint rule, Trapezoidal rule, or Simpson's rule. Use $n = 6$. Compare your approximation to the exact value.

Exact: $\int_0^{\pi/2} x \sin x\, dx$

$u = x \quad dv = \sin x$

$du = dx \quad v = -\cos x$

Indef Integral $\Rightarrow -x\cos x + \int \cos x\, dx = -x\cos x + \sin x + C$

$$\int_0^{\pi/2} x\sin x = \sin x - x\cos x \Big|_0^{\pi/2}$$

$$= \sin\frac{\pi}{2} - \frac{\pi}{2}\cancel{\cos\frac{\pi}{2}} - \cancel{\sin 0} - 0\cancel{\cos 0}$$

$$= \sin\frac{\pi}{2}$$

$$= 1$$