1 a) $\int \frac{\cos(1/x)}{x^2}\,dx = \int \frac{\cos u}{x^2}\,dx = \int \cos u\,du = -\sin u + c = -\sin\left(\frac{1}{x}\right) + c$

Let $u = \frac{1}{x}$

$du = -\frac{1}{x^2}dx$

$-du = \frac{1}{x^2}dx$

b) $\int \frac{e^{(\arctan x)}}{1+x^2}\,dx = \int e^u\,du = e^u + c = e^{\arctan x} + c$

Let $u = \arctan x$ $du = \frac{1}{1+x^2}dx$

2. $v(t) = t^2e^t$ $s(40) - s(0) = \int_0^{40} v(t)\,dt = \int_0^{40} t^2e^t = t^2e^t - 2te^t + 2e^t \Big|_0^{40} = e^{40}(40^2 - 80 + 2) - 2 = (1522e^{40} - 2)$

IBP, Let $u = t^2$; $du = 2t\,dt$; $dv = e^t dt$; $v = e^t$

IBP$_2$ Let $u = 2$; $du = dt$; $dv = e^t dt$; $v = e^t$

3. $\int_0^1 \sqrt[3]{t^3+1} \cdot t^5\,dt = \int_0^1 \sqrt[3]{t^3+1} \cdot t^2 \cdot t^3\,dt = \frac{1}{3}\int_1^2 (u-1)u^{1/3}\,du = \frac{1}{3}\int_1^2 u^{4/3} - u^{1/3}\,du = \frac{1}{3}\left[\frac{(t^3+1)^{4/3+3/3}}{7/3} - \frac{(t^3+1)^{1/3+3/3}}{4/3}\right]_1^2 =$

Let $u = t^3+1$ $du = 3t^2\,dt \rightarrow \frac{1}{3}du = t^2\,dt$

$\frac{1}{3}\left[\frac{3}{7}u^{7/3} - \frac{3}{4}u^{4/3}\right]_1^2 = \frac{1}{7}(2^{7/3}) - \frac{1}{4}(2^{4/3}) - \frac{1}{7} + \frac{1}{4} = \frac{4\sqrt[3]{2}}{7} - \frac{\sqrt[3]{2}}{2} - \frac{1}{7} + \frac{1}{4}$

$\frac{8\sqrt[3]{2}}{14} - \frac{7\sqrt[3]{2}}{14} + \frac{3}{28} = \frac{\sqrt[3]{2}}{14} + \frac{3}{28} = \frac{2\sqrt[3]{2}+3}{28} \approx 0.1971$

4. $\int_e^{e^2} \frac{1}{x(\ln x)^2}\,dx = \int_1^2 u^{-2}\,du = -\frac{1}{u}\Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$

Let $u = \ln x$ $du = \frac{1}{x}dx$

5. $\int_0^1 \frac{e^x}{e^x+1}\,dx = \int_2^{e+1} \frac{1}{u}\,du = \ln|u|\Big|_2^{e+1} = \ln(e+1) - \ln(2) = \ln\left(\frac{e+1}{2}\right)$

Let $u = e^x + 1$; $du = e^x\,dx$

6. $\int_{\pi/4}^{\pi/3} \frac{\sin x}{\sqrt{1-\cos x}}\,dx = \int_{1-1/\sqrt{2}}^{1/2} \sin x\, u^{-1/2}\,dx = \int_{1-1/\sqrt{2}}^{1/2} u^{-1/2}\,du = 2u^{1/2} = 2\left[\sqrt{1/2} - \sqrt{1-\frac{1}{\sqrt{2}}}\right]$

Let $u = 1 - \cos x$ $du = \sin x\,dx$

7. $\int \sin^3 x\,dx = \int \sin^2 x \sin x\,dx = \int (1-\cos^2 x)\sin x\,dx = -\int(1-u^2)\,du = \int(u^2-1)\,du = \frac{1}{3}u^3 - u + c = \frac{1}{3}(\cos x)^3 - \cos x + c$

$u = \cos x$ $du = -\sin x\,dx$ $-du = \sin x\,dx$

8. $\int_0^{\pi} x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x\Big|_0^{\pi} = \pi\sin\pi + \cos\pi - 0\sin 0 - \cos 0 = 0 + -1 - 0 - 1 = -2$

Let $u = x$ $du = dx$ $dv = \cos x\,dx$ $v = \sin x$

9. $\int_0^1 x\tan^{-1}x\,dx = \frac{x^2}{2}\tan^{-1}x\Big|_0^1 - \frac{1}{2}\int_0^1 \frac{x^2}{1+x^2}\,dx = \frac{x^2}{2}\tan^{-1}x\Big|_0^1 - \frac{1}{2}\int_0^1\left(1 - \frac{1}{1+x^2}\right)dx = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x\Big|_0^1 = \left(\frac{1}{2}\cdot\frac{\pi}{4}\right) - \frac{1}{2} +$

Let $u = \tan^{-1}x$ $du = \frac{1}{1+x^2}dx$ $dv = x\,dx$ $v = \frac{1}{2}x^2$ $1+x^2\overline{)x^2+0x+0}$

$\frac{2\pi}{8} - \frac{1}{2} = \frac{\pi}{4} - \frac{1}{2}$

10. $\int_{-\infty}^{\infty} xe^{-x^2}\,dx = \int_{-\infty}^{0} xe^{-x^2}\,dx + \int_0^{\infty} xe^{-x^2} = -\frac{1}{2} + \frac{1}{2} = 0$

$\int_0^{\infty} xe^{-x^2} = \lim_{b\to\infty}\int_0^b xe^{-x^2}\,dx = \lim_{b\to\infty} -\frac{1}{2}e^{-x^2}\Big|_0^b = \lim_{b\to\infty}\left[-\frac{1}{2}e^{-b^2} + \frac{1}{2}\right] = \frac{1}{2}$

Let $u = -x^2$ $du = -2x\,dx$ $x\,dx = -\frac{1}{2}du$ $\int xe^{-x^2}\,dx = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + c = -\frac{1}{2}e^{-x^2}$

$\int_{-\infty}^{0} xe^{-x^2}\,dx = \lim_{a\to-\infty}\int_a^0 xe^{-x^2}\,dx = \lim_{a\to-\infty}\left[-\frac{1}{2}e^{-x^2}\right]_a^0 = \lim_{a\to-\infty}\left[-\frac{1}{2} + \frac{1}{2}e^{-a^2}\right] = -\frac{1}{2}$

11. $\int \frac{1}{x^2-16}\,dx = \int \frac{1}{(x+4)(x-4)}\,dx = \frac{1}{8}\int\left(\frac{1}{x-4} - \frac{1}{x+4}\right)dx = \frac{1}{8}\left[\ln|x-4| - \ln|x+4|\right] + c = \frac{1}{8}\ln\frac{|x-4|}{|x+4|} + c = \ln\sqrt[8]{\frac{|x-4|}{|x+4|}} + c$

PFD $\frac{1}{(x+4)(x-4)} = \frac{A}{x+4} + \frac{B}{x-4} = -\frac{1}{8}\left(\frac{1}{x+4}\right) + \frac{1}{8}\left(\frac{1}{x-4}\right)$

$A = -1/8$ $B = 1/8$

12. $\int \frac{dx}{x^2\sqrt{4-x^2}} = \int \frac{2\cos\theta\,d\theta}{4\sin^2\theta\sqrt{4-4\sin^2\theta}} = \int \frac{2\cos\theta\,d\theta}{4\sin^2\theta\sqrt{4(1-\sin^2\theta)}} = \int \frac{2\cos\theta\,d\theta}{4\sin^2\theta \cdot 2\sqrt{\cos^2\theta}} = \int \frac{2\cos\theta\,d\theta}{4\sin^2\theta\,2\cos\theta} = \int \frac{d\theta}{4\sin^2\theta} = \frac{1}{4}\int \csc^2\theta\,d\theta = -\frac{1}{4}\cot\theta + c$

$= -\left(\frac{\sqrt{4-x^2}}{4x}\right) + c$

Let $x^2 = 4\sin^2\theta$; $x = 2\sin\theta$; $dx = 2\cos\theta\,d\theta$

13. y', $y = \int_2^{\sec x} \frac{dt}{\sqrt{t^2-1}}$ $y' = \frac{\sec x\tan x}{\sqrt{\sec^2 x - 1}} = \frac{\sec x\tan x}{\tan x} = \sec x$