Name ______________________________ Calculus 2 Spring 2007 Exam Mike Huff

Do all work on the test paper for partial credit. Good luck!!

(12 points)

1. Determine whether each sequence converges or diverges, giving a reason, and if it does converge, find its limit.

a) $a_n = \dfrac{(-1)^n}{\sqrt{n+1}}$

b) $a_n = (-1)^n\left(1 - \dfrac{1}{\sqrt{n}}\right)$

c) $a_n = \dfrac{5\cos n + n}{n^2}$

d) $a_n = \dfrac{\ln(n^2)}{n}$

e) $a_n = \left(\dfrac{n-2}{n}\right)^n$

f) $a_n = \left(\dfrac{n+1}{n-1}\right)^n$

(6 points)

2. Consider the sequence defined by $a_n = \left(\frac{2}{3}\right)^n$

a) Write the first five terms of the sequence.

b) Determine the limit of the sequence.

c) Let $b_n = \frac{a_{n+1}}{a_n}$. Write the first five terms of this sequence.

d) Determine the limit of b_n.

e) Let $c_n = \sum_{k=1}^{n} a_k$. Write the first five terms of c_n.

f) Determine the limit of c_n.

(10 points)

3. Determine whether each series is convergent or divergent. If it is convergent, find its sum.

a) $-\frac{81}{100}+\frac{9}{10}-1+\frac{10}{9}-\cdots$

b) $\sum_{n=1}^{\infty}\cos\left(\frac{\pi}{2n^2-1}\right)$

c) $\sum_{n=0}^{\infty}\left(\frac{2^{n+1}}{5^n}\right)$

d) $\sum_{n=0}^{\infty}\frac{4}{(4n-3)(4n+1)}$

e) $\sum_{n=1}^{\infty}e^{-2n}$

(6 points)

4. Determine whether each series is convergent or divergent. Explain.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + \ln n}$

b) $\displaystyle\sum_{n=2}^{\infty} \frac{3^n + 5^n}{15^n}$

(6 points)

5. Find a closed-form expression for the nth partial sum of the series $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

Hint: Write out the first few partial sums.

(6 points)

6. For what value or values of a, if any, does the series $\sum_{n=1}^{\infty}\left(\frac{a}{n+1}-\frac{1}{n+4}\right)$ converge?

Carefully explain your reasoning for full credit.

(6 points)

7. There is absolutely no empirical evidence for the divergence of the harmonic series even though we know that it diverges. The partial sums, which satisfy the inequality

$$\ln(n+1) \le \int_1^{n+1} \frac{1}{x}dx \le 1+\frac{1}{2}+\cdots+\frac{1}{n} \le 1+\int_1^{n}\frac{1}{x}dx = 1+\ln(n),$$

just grow too slowly. To see what I mean, suppose you had started with $s_1 = 1$ the day the universe was formed, thirteen billion years ago, and added a new term every *second*. About how large would s_n be today?

(9 points)

8. Determine whether each series converges absolutely, converges conditionally, or diverges. Carefully explain your reasoning for full credit.

a) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$

c) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$

(7 points)

9. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n2^n}$.

(8 points)

10. Find a power series representation for the function $f(x) = \frac{1-x}{1+x}$ and find its radius of convergence.

(8 points)

11. Power Series

a) Express $\int \frac{1}{1+x^5}\,dx$ as a power series.

b) Express e^{x^2} as a Taylor series. What is its radius of convergence?

(8 points)

12. Find the Taylor polynomial of degree 5 at $x = 0$ for the function defined by $f(x) = \sin x$. Then compute the value of $\sin\left(\frac{\pi}{6}\right)$ to as many decimal places as the polynomial of degree 5 allows.

(8 points)

13. Find the 3rd-degree Taylor polynomial centered at $x = 4$ for the function $f(x) = \sqrt{x}$. Use the polynomial to approximate $\sqrt{6}$. Find a bound on the error in your approximation.

Calculus 2
Math 2414
Test 4
Spring 2007
Mike Huff

You may discuss this test with others in the class but you may not use *any* other outside resources (that includes the Learning Lab.)