Name ______________________________ Calculus 2 Summer 2011 Huff Test 4

(8 points)

1. A function $y(x)$ satisfies the differential equation $\frac{dy}{dx} = k(8 - y^2)$, $k > 0$ is a positive constant.

a) What are the constant (equilibrium) solutions of the equation?

b) For what values of y is y increasing?

c) For what values of y is y decreasing?

d) Classify all equilibria as stable or unstable.

(8 points)

2. Solve the differential equation $\frac{dy}{dt} = 2t\sqrt{1-y}$ subject to the initial condition $y(1) = 0$.

(8 points)

3. Solve the differential equation $y' = -2y$ subject to the initial condition $y(0) = 3$. From your solution, find the value of $y(5)$.

(8 points)

4. Solve the differential equation $\frac{dy}{dt} = 2ty^2 + 3y^2$ subject to the initial condition $y(0) = 1$.

(8 points)

5. Find the solution of the initial value problem $\frac{dy}{dx} = 3y + 1$, $y(0) = 2$.

(10 points)

6. Find the solution of the initial value problem $\frac{dy}{dx} = 5y(1000 - y)$,

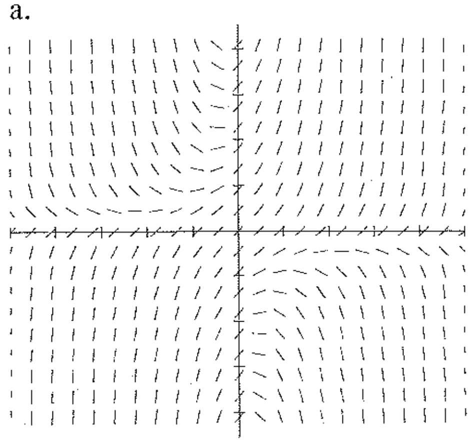
$y(0) = 500$.

(5 points)

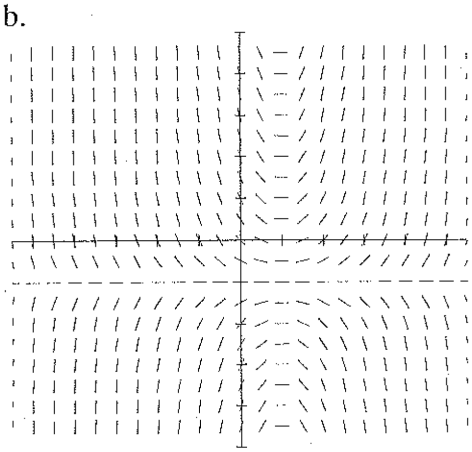
7. Identify the direction field for the given differential equation and then make a sketch of the solution with the given initial value on the correct direction field.

$$y' = (x - 1)(y + 1), \qquad y(0) = 2$$

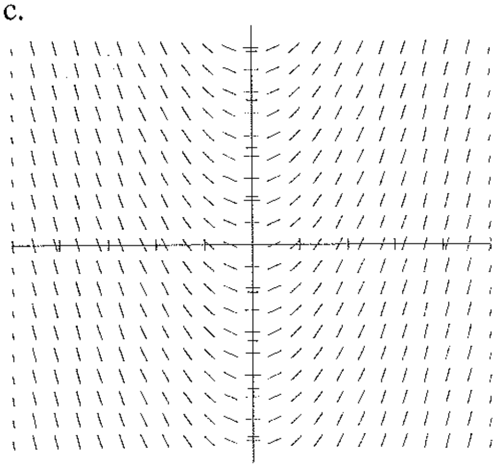
a.



b.



c.



(10 points)

8. Newton's Law of cooling is described by the differential equation $\frac{dT}{dt} = k(T - T_a)$. Suppose an object has an initial temperature of 250° F and that after one hour in a room that is 72° F the temperature has lowered to 200° F. Find the temperature of the object at any time t.

(10 points)

9. A tank contains 500 L of brine with 10 kg of dissolved salt. Brine containing 0.05 kg of salt per liter enters the tank at a rate of 8 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate.

   a) Find the amount of salt in the tank after t minutes?

   b) How much salt is in the tank after 30 minutes?

   c) As the time increases without bound, what happens to the amount of salt in the tank?

   d) What is the concentration then?

(10 points)

10. A tank contains 475 liters of brine with 12 kg of dissolved salt. Pure water enters the tank at a rate of 15 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate.

a) Find the amount of salt in the tank after t minutes?

b) How much salt is in the tank after 15 minutes?

c) As the time increases without bound, what happens to the amount of salt in the tank?

d) What is the concentration then?

(10 points)

11. In the absence of air resistance for an object in free fall, the velocity is the solution of the differential equation $\frac{dv}{dt} = g$ where g is the constant of acceleration due to the earth's gravity.

a) Find the solution of this differential equation if $v(0) = v_0$.

b) As the time increases without bound, what happens to v?

(5 points)

12. Find the slope of the tangent line to the graph of the function $f(x)$ defined by $f(x) = \int_0^x \sec(t)dt$ at $x = \frac{\pi}{4}$