College Algebra Test 2 Review Mike Huff

Graphs and the Cartesian Coordinate System Part 2

<u>The Midpoint Formula</u>: Given any two points (x_1, y_1) and (x_2, y_2)**,** the point M halfway between the two points can be found by the **Midpoint Formula:**

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Exercise 2.1:

a) Find the **<u>distance</u>** between the points (-3, -5) and (7,4).
b) Find the **<u>midpoint</u>** of the line segment joining the same two points.
c) Find the **<u>slope</u>** of the line joining the same two points.
d) A circle has a diameter with endpoints $(-1, 3)$ and $(5, -9)$. Find the equation of the circle.

Facts about Slope:

1. The slope of a horizontal line is zero.
2. A line that rises going from left to right has positive slope.
3. A line that falls going from left to right has negative slope.
4. The slope of a vertical line does not exist. Why?
5. Two lines are **parallel** lines if they have the same slope.
6. Two lines are **perpendicular** lines if the product of the slopes is -1. That is $m_1 \cdot m_2 = -1$

<u>Forms of Equations of Lines:</u>

Slope-intercept: $y = mx + b$
Point-slope: $y = m(x - x_1) + y_1$
Horizontal: $y = b$
Vertical: $x = a$

Exercise 2.2:

a) Write an equation for the line that passes through the points (-2,5) and (4, 5).
b) Write an equation for the line that passes through the points (-2,5) and (-2, -2).
c) Write an equation for the line that passes through the points (-2,5) and (4,-5).
d) Write an equation of the line parallel to the line $-3x + 5y = 11$ and passing through $(-2, 3)$
e) Write an equation of the line perpendicular to the line $-3x + 5y = 11$ and passing through $(-2, 3)$.

Linear Equations

Exercise 2.3: Solving Linear Equations in One Variable

a) $6x + 3 = 3x - 11$
b) $f(x) = 3(x-3) + 2(x-6)$ Solve for $f(x) = 0$
c) $7(x+1) - 4(2x+3) = 3(x+1)$
d) $\frac{t}{2} - \frac{t}{5} = 4$
e) $\frac{5x}{2} + \frac{3}{4}(x-4) = 0$
f) $\frac{10-4x}{4} = \frac{5x+6}{3} + 2$
g) $\frac{7x+3}{4x+1} = \frac{1}{3}$

Solving Linear Inequalities

Exercise 2.4: Solving Inequalities Symbolically and Graphically

Solve the following inequalities symbolically and then check your solution graphically.

a) $-2x + 5 > 7$

b) $5(2-x) > 7(x-1)$

c) $\frac{x}{3} + \frac{3}{5} \leq -1$

d) $\frac{2x-5}{3} \geq \frac{3x-2}{5}$

e) $3 < \frac{5x-1}{3} \leq 15$

Piecewise-Defined Functions

Functions defined in pieces are called **piecewise-defined functions**.

Exercise 2.5: For each of the following piecewise-defined functions answer the questions below:

i. What is the domain of *f*?

ii. Evaluate at $x = -3, 2, 4, 5, 6$.

iii. Sketch a graph

iv. Is *f* continuous?

v. Find the values where $f(x) = -4$

a) Let $f(x) = \begin{cases} x-2 & -5 \le x < 2 \\ -3x & 2 \le x \le 5 \end{cases}$

b) $f(x) = |x| = \begin{cases} -x & x < 0 \\ x & x \ge 0 \end{cases}$

c) $f(x) = \begin{cases} -3x & -3 < x \le 1 \\ -3 & 1 < x < 3 \\ x-6 & 3 \le x < 5 \end{cases}$

d) $f(x) = \begin{cases} 2x-5 & x \le 1 \\ x-4 & x > 1 \end{cases}$

Absolute Value Functions

Exercise 2.6: Analyzing the graph of $y = |ax+b|$

For each function *f*, graph $y = f(x)$ and $y = |f(x)|$ separately.

a) $f(x) = x + 5$

b) $f(x) = -3x + 3$

Solving Absolute Value Equations

To solve the equation $|ax+b| = c$, set $ax + b = \pm c$ and solve both equations.

Exercise 2.7: Solving Absolute Value Equations

a) $|x| = 5$
b) $|x-1| = 5$
c) $|3x-1| = 7$

Solving Absolute Value Inequalities

Let the inequality $|x| < 5$ be given. If we graph the functions $f(x) = |x|$ and $g(x) = 5$, we see that $f(x) < g(x)$ precisely when x is less than 5 units from zero. That is,

$$\text{If } |x| < 5 \text{ then } -5 < x < 5$$

$$\text{If } |x| \leq 5 \text{ then } -5 \leq x \leq 5$$

If $|x| > 5$ then x must be more than 5 units from zero. That is,

$$\text{If } |x| > 5 \text{ then } x < -5 \text{ or } x > 5$$

$$\text{If } |x| \geq 5 \text{ then } x \leq -5 \text{ or } x \geq 5$$

> Let the solutions of $|ax + b| = c$, $c > 0$ be s_1 and s_2 with $s_1 < s_2$. Then, the solution set of $|ax + b| < c$ is $s_1 < x < s_2$ and the solution set of $|ax + b| > c$ is $x < s_1$ or $x > s_2$.

Exercise 2.8: Solve graphically and symbolically

a) $|x| < 9$

b) $|x - 2| < 5$

c) $|2x - 7| \leq 11$

d) $|x| > 4$

e) $|x - 2| > 7$

f) $|3x - 2| \geq 11$

g) $|1 - 3x| \geq 4$

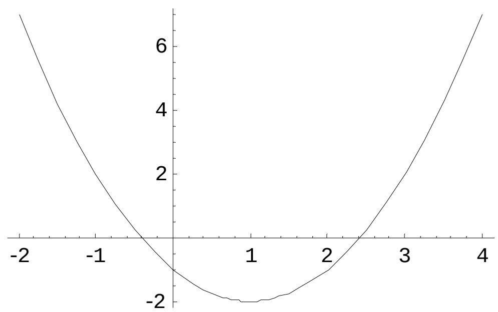
Quadratic Functions and Their Graphs

Definition: The function $f(x) = ax^2 + bx + c$, $a \neq 0$ is called a **quadratic function**. The form $f(x) = ax^2 + bx + c$ is called the **standard form**.

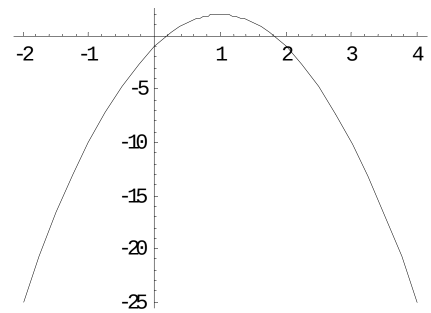
The graphs of quadratic functions have a distinct shape called a **parabola**.

Examples:

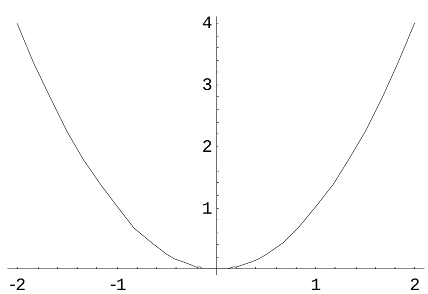
a) $f(x) = x^2 - 2x - 1$



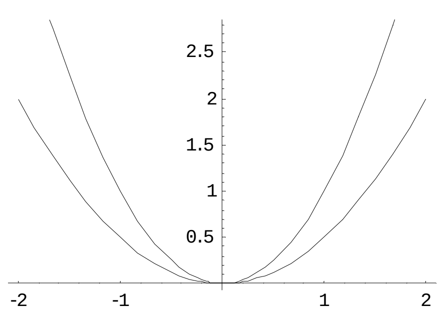
b) $f(x) = -x^2 - 2x - 1$

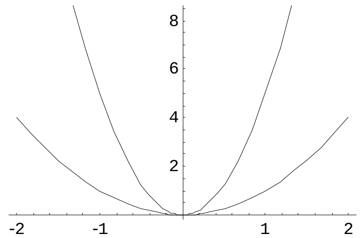


The parameter *a* determines whether the graph opens up or opens down. In a) above $a > 0$ so the graph opens upward. In b) above $a < 0$ so the graph opens downward. The value of *a* also changes the width of the graph. If $a = 1$, then the graph looks like this:



The value of *a* also changes the width of the graph. If $a < 1$, then the graph is wider:



If $a > 1$, then the graph is narrower: 

Definition: If a function has a graph that is a parabola, then the point where the graph achieves its maximum or minimum value is called the **vertex** of the parabola**.** The *x*-coordinate of the vertex is found using the formula $x = \frac{-b}{2a}$. The *y*-coordinate of the vertex is given by $f\left(\frac{-b}{2a}\right)$. If the parabola opens upward then the function will have a minimum value at the vertex. If the parabola opens downward then the function will have a maximum value at the vertex.

Definition: The **vertex form** of a quadratic function is $f(x) = a(x-h)^2 + k$. The vertex of the parabola is (h,k). The axis of symmetry is the vertical line $x = h$.

Exercise 2.9:

a) Write the function $g(x) = -(x-2)^2 + 1$ in standard form.
b) Write the function $g(x) = -\frac{1}{2}(x-3)^2 - 2$ in standard form.
c) Write the function $f(x) = x^2 + 4x - 2$ in vertex form and identify the vertex.
d) Write the function $F(x) = x^2 + 3x + \frac{1}{4}$ in vertex form and identify the vertex.
e) Write the function $h(x) = -2x^2 + 4x - 3$ in vertex form and identify the vertex.

Exercise 2.10:

a) Find the equation of the function that has vertex at $(-3,5)$ and whose graph passes through the point $(-6,-1)$
b) Find the equation of the quadratic function that has vertex at $(2,-3)$ and whose graph passes through the point $(-2,3)$
c) Find a function $f(x) = a(x-h)^2 + k$ that models the data exactly:

x	2	3	4	5
f(x)	4	7	16	31

Quadratic Equations

Definition: An equation is a quadratic equation if it can be written in the form

$$ax^2 + bx + c = 0.$$

Factoring

One way to solve quadratic equations is by factoring. This method makes use of the Zero Product Property.

Theorem: Zero Product Property

If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$ or both a and b are zero.

Exercise 2.11: Solve the following quadratic equations.

a) $(x-5)(x+2) = 0$
b) $x^2 + 5x + 6 = 0$
c) $x^2 - 5x = 14$

You will need to remember the **special product formulas:**

$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$
$$(x+y)(x-y) = x^2 - y^2$$

and

$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$
$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

Exercises 2.12: Solve the following quadratic equations by factoring.

a) $x^2 - 16 = 0$
b) $2x^2 - x = 3$
c) $x^2 - 3x - 4 = 0$
d) $6x^2 + 7x - 3 = 0$
e) $6x^2 + 11x = 10$
f) $6 + \frac{13}{x} + \frac{6}{x^2} = 0$

The Square Root Property

Simple quadratic equations of the form $x^2 = k$ can be solved using the square root property.

Theorem: The Square Root Property. If $x^2 = k$ and k is any real number, then $x = \pm\sqrt{k}$.

Notice that, if $k < 0$, the two solutions of the equation are complex conjugate pairs.

Example: Solve the quadratic equation $x^2 - 16 = 0$ using the square root property.

First rewrite the equation as $x^2 = 16$. By the square root property, the equation has the solutions $x = \pm\sqrt{16} = \pm 4$. Checking both solutions, we see that they both solve the original equation: $4^2 - 16 = 0$ is true, and

$(-4)^2 - 16 = 0$ is also true.

Exercise 2.13: Solve the following quadratic equations using the square root property.

a) $x^2 + 10 = 60$
b) $x^2 - 1 = 0$
c) $(x-5)^2 = 20$
d) $2(x+3)^2 + 7 = 17$
e) $3(x-5)^2 - 7 = 12$

f) The function $P(x) = 0.00048(x-1899)^2 + 1.291$ can be used to model the world population where x is the year and $P(x)$ is the world population in billions. Use the model to estimate when the world population might reach 10 billion.

Completing the Square

The process of completing the square is related to perfect square trinomials, that is, to trinomials of the form

$$a^2 + 2ab + b^2 = (a+b)^2, \text{ or } a^2 - 2ab + b^2 = (a-b)^2.$$

1. The first step when completing the square will always be to divide both sides of the equation by the coefficient of the x^2 term to make the leading coefficient one.
2. The second step is to use the addition property of equality to move the constant to the right-hand side.
3. Take one-half the coefficient of the 1st-degree or x term, square it, and add the result to both sides of the equation using the addition property of equality.
4. Factor the left-hand side using the fact that we have constructed a perfect square trinomial.
5. Use the square root property to solve the resulting equation:
6. Check your answers in the original equation.

Exercise 2.14: Solve by completing the square.

a) $x^2 - 9x - 14 = 0$
b) $x^2 - 2x - 4 = 0$
c) $-x^2 - 2x + 4 = 0$
d) $-x^2 - 3x + 7 = 0$
e) $2x^2 - 7x + 4 = 0$
f) $4x^2 - 12x + 9 = 0$

One of our main goals in introducing completing the square was to use the technique to solve the general quadratic equation $ax^2 + bx + c = 0$. Solving the general form of this equation by completing the square will produce a formula to solve any quadratic equation in terms of the constants *a*, *b*, and *c*. This result is called the **quadratic formula.**

The Quadratic Formula.

The Quadratic Formula: If $ax^2 + bx + c = 0$, and $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula gives us a way of solving all second-degree equations in one variable. Remember that the square root of a negative number is an imaginary number. This means that if the quantity $b^2 - 4ac$ is negative, then the equation $ax^2 + bx + c = 0$ will have two complex conjugate solutions. This important result about the **discriminant,** $b^2 - 4ac$**,** and others are summarized in the table below:

$b^2 - 4ac > 0$	Two real solutions
$b^2 - 4ac = 0$	One real solution (called a double root)
$b^2 - 4ac < 0$	Two complex conjugate pair solutions

Solving Equations Using the Quadratic Formula

To solve an equation using the quadratic formula you must follow these steps:

1) Write the equation in standard form, $ax^2 + bx + c = 0$, and determine the numerical values of a, b, and c.

2) Substitute the values for a, b, and c into the quadratic formula and then evaluate the formula to obtain the solution(s).

Exercise 2.15: Solve the following equations using the quadratic formula.

a) $x^2 - 6x = -5$
b) $x^2 - 6x = 0$
c) $3x^2 - 4x - 5 = 0$
d) $-6x^2 + 5x + 5 = 0$
e) $x^2 - \frac{1}{5}x - \frac{1}{3} = 0$

Exercise 2.16: Applications of Quadratic Equations

a) A square television set has a diagonal that is 32". How long is each side of the set?

b) Use the cost equation $C = 0.5x^2 + 15x + 5000$ to find the number of units that can be produced for total costs $C = \$12,000$.

c) The height s, in feet, of a projectile above the ground for any time, t, can be approximated using the formula $s = -16t^2 + v_0 t + s_0$, where v_0 is the initial velocity and s_0 is the projectiles initial height. If a projectile is launched from the ground with an initial velocity of 320 feet per second, its height is described by $s = -16t^2 + 320t$. If the projectile is launched at time $t = 0$, when does it return to the ground?

d) The path of a diver is given by $y = -0.44x^2 + 2.67x + 10$ where y is the height in feet and x is the horizontal distance from the end of the diving board in feet. Find the maximum height of the dive.

e) A company is going to produce a new product. Its marketing department has estimated that the revenue for the new product will be given by the equation

$$R(x) = 500x - 0.0005x^2.$$

Where x is the number of units produced and R is the revenue measured in dollars. Find the following:

  a) the number of units that produces a maximum revenue
  b) the maximum revenue

f) A baseball is hit 3 feet above the ground at a velocity of 100 feet per second and at an angle of 45° with respect to the ground. The path of the baseball is given by the function $f(x) = -0.0032x^2 + x + 3$ where $f(x)$ is the height of the baseball (in feet) and x is the distance from home plate (in feet).

  a) What is the maximum height reached by the baseball?
  b) How far is the ball from home plate when it hits the ground?

g) A manufacturer determines that the variable costs for a new product are \$3.17 per unit and the fixed costs are \$75,000. The product is to be sold for \$5.25. Let x be the number of units sold.

  a) Write the total cost C as a function of the number of units sold
  b) Write the total revenue R as a function of the number of units sold.
  c) Write the total profit P as a function of the number of units sold.

h) A projectile is fired straight upward from the ground with an initial velocity of 258 feet per second. When will its height be 300 feet? (Hint: Use the formula $s = -16t^2 + v_0 t + s_0$)

i) In the United States, post secondary degrees below bachelor's degrees earned during the years 1985 to 1990 can be approximated by the model

$$f(x) = 5334.59x^2 - 23024x + 617519.11$$

where $x = 0$ corresponds to 1985. Use the graph of this function in the window $[0,6,1]$ by $[500000, 800000, 50000]$ to determine the year during this interval that the number of these degrees earned reached a minimum. Support your work symbolically by putting the function into vertex form.

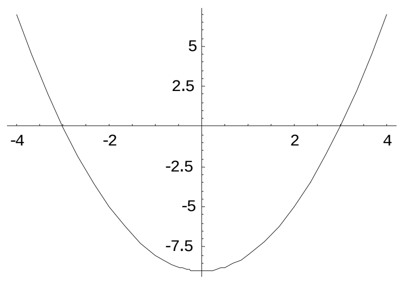
Quadratic Inequalities

Definition: A quadratic inequality is an inequality of the form $ax^2 + bx + c > 0$.

Example: Graphical Solution of Quadratic Inequalities: Solve the inequalities graphically using the graph below.

a) $x^2 - 9 > 0$

b) $x^2 - 9 < 0$



Example: Symbolic Solution of Quadratic Inequalities: Solve the inequality graphically $x^2 + 5x + 6 > 0$ symbolically.

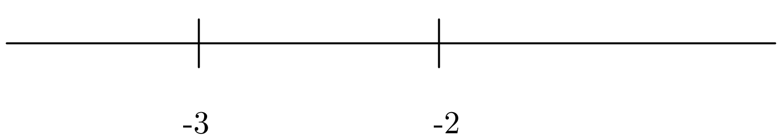
To solve a quadratic inequality symbolically, we do the following:

1. Put the inequality in standard form, i.e. $P > 0$.
2. Find the zeros (x-intercepts) of the polynomial. These are called the **critical numbers**.
3. Plot the critical numbers on a number line. These numbers determine test intervals.
4. Choose a test number in each test interval and determine whether the inequality is true or false on that interval.
5. Write the solution using interval notation.

To solve $x^2 + 5x + 6 > 0$, first, set $x^2 + 5x + 6 = 0$ and solve. Since,

$x^2 + 5x + 6 = (x+3)(x+2) = 0$, we have $x = -3, -2$ as the solutions.

Next, we plot the solutions (zeros) on the number line to form three test intervals. Pick a test number in each interval and determine if that number makes the inequality true or false.



$(-\infty, -3)$	$(-3, -2)$	$(-2, \infty)$	Test Interval
-4	-2.5	0	Test Number
True	False	True	True? or False?

The solution is the union of all the intervals for which the inequality is true. The solution is $(-\infty,-3)\cup(-2,\infty)$

Exercise 2.17:

a) $x^2-8x+7<16$ Solve graphically, numerically, and symbolically.

b) The function given by $f(x)=0.000478x^2-1.813x+1720$ models the world population in billions where x is the year. Determine the years that correspond to world population between 3 and 5 billion, inclusively.

c) The table shows the average price of a movie ticket for several years.

Year (x)	1950	1960	1980
Cost $)	0.50	0.70	2.50

i. Find the values of $a, h, \text{and} k$ so that $f(x)=a(x-h)^2+k$ models the data, where *x* represents time. (Hint: Let $(1950, 0.50)$ be the vertex.)

ii. Evaluate $f(1975)$ and interpret the result.

iii. Estimate the years when the cost of a movie ticket will be between $10 and $11.

d) $*P* invested at an interest rate *r* compounded annually increases to an amount given by $A=P(1+r)^2$ in 2 years. If an investment of $3,000 is to increase to an amount greater than $3,500 in 2 years, the interest rate must be greater than what percent?

e) The revenue and cost equations for a product are given by

$$C=15x+75{,}000 \text{ Cost equation}$$

$$R=x(77-0.0008x) \quad \text{Revenue equation}$$

Where *R* and *C* are measured in dollars and *x* represents the number of units sold. How many units must be sold to obtain a profit of at least $100,000?

f) A projectile is fired straight upward from the ground with an initial velocity of 258 feet per second. During what time interval will its height exceed 300 feet? (Hint: Use the formula $s=-16t^2+v_0t+s_0$)

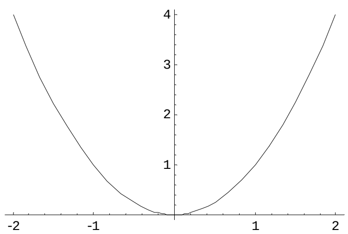
g) The revenue and cost equations for a product are given by $R=x(75-0.0006x)$ and $C=25x+25{,}000$, respectively. Where *R* and *C* are measured in dollars and *x* represents the number of units sold. How many units must be sold to obtain a profit of at least $100,000?

Transformations

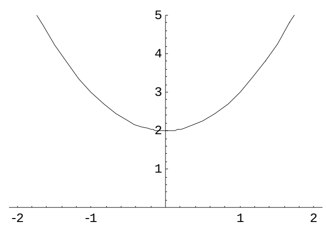
The transformations that do not change the shape of the graph are called **rigid transformations**. The transformations that change the shape of the graph are called **non-rigid transformations**.

Horizontal and Vertical Translations

Definition: For any real number, h, the graph of $f(x+h)$ is a **horizontal translation** of the graph of $f(x)$; It has the same shape but has been moved h units to the left or right depending upon the sign of h. For any real number, k, the graph of $f(x)+k$ is a **vertical translation** of the graph of $f(x)$; It has the same shape but has been moved k units up or down depending upon the sign of k.

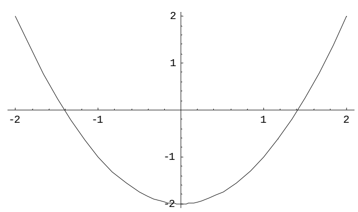


$f(x) = x^2$



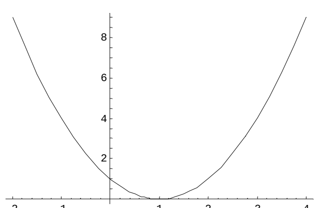
$g(x) = x^2 + 2$

$y = f(x) - k$ is a vertical shift of the graph downward *k* units.

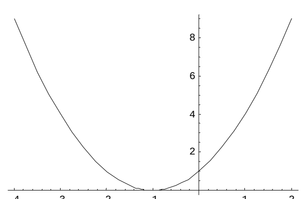


$g(x) = x^2 - 2$

$y = f(x+h)$ is a horizontal shift of the graph left *h* units.
$y = f(x-h)$ is a horizontal shift of the graph right *h* units.



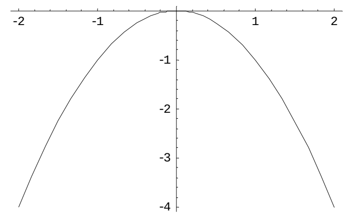
$g(x) = (x-1)^2$



$g(x) = (x+1)^2$

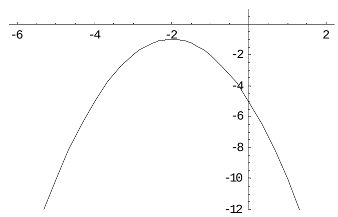
Reflection

$y=-f(x)$ is a reflection of the graph of *f* across the *x*-axis.



$g(x)=-x^2$

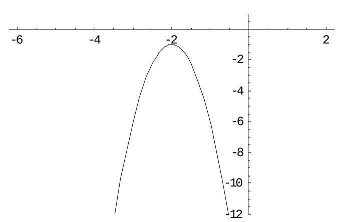
You can combine the above transformations:



$g(x)=-(x+2)^2-1$

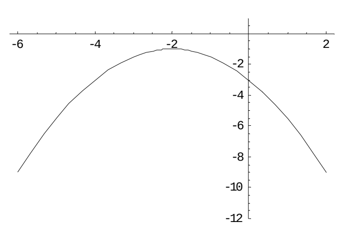
Expansion and Contraction

$y=Cf(x)$ is an expansion (vertical stretching) of the graph of *f* if $C>1$.



$g(x)=-5(x+2)^2-1$

$y=Cf(x)$ is a contraction (vertical shrinking) of the graph of *f* if $0<C<1$.



$$g(x)=-\frac{1}{2}(x+2)^2-1$$

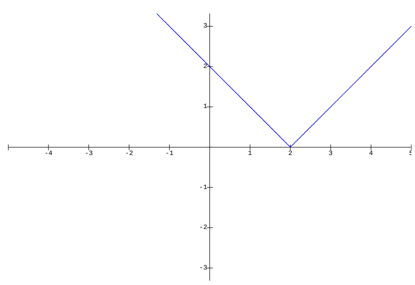
Exercise 2.18: Transformations

1. How are the following graphs related to the graph of $f(x) = x^2$?
   List all the transformations that apply.

   a) $f(x) = x^2 - 4$ ________________________________
   b) $f(x) = 4(x+4)^2 - 4$ ________________________________
   c) $f(x) = (x-4)^2$ ________________________________
   d) $f(x) = -\frac{1}{4}x^2 + 4$ ________________________________
   e) $f(x) = -4x^2 - 4$ ________________________________

2. Put the quadratic function $f(x) = x^2 + 2x - 1$ in vertex form and graph.
   a) What transformations are used to transform the graph of $g(x) = x^2$ into this graph?
   b) What is the axis of symmetry of the graph of *f*?
   c) What is the vertex of the graph? Label this clearly
3. Given the graph of *f*(*x*) below, sketch graphs of the following functions

   a) $g(x) = f(x-1)$ b) $g(x) = f(x) - 1$



4. Given the table of values defining *f*(*x*) below, fill in the table of values for the following functions

x	2	3	4	5
f(x)	4	7	16	31

a) $g(x) = f(x-1)$

x				
g(x)				

b) $h(x) = f(x) - 1$

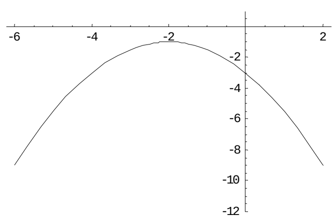
x				
h(x)				

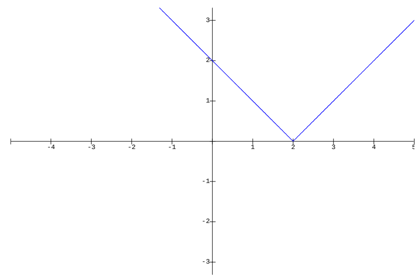
More about Functions

Definitions: Let *I* be an interval in the domain of a function *f*. Then:

1. *f* is **increasing** on *I* if whenever $a, b \in I$ and $a < b$, we have $f(a) < f(b)$.
2. *f* is **decreasing** on *I* if whenever $a, b \in I$ and $a < b$, we have $f(a) > f(b)$.
3. *f* is **constant** on *I* if we have $f(a) = f(b)$ for all $a, b \in I$.

Exercise 2.19: Use the graph to determine intervals where the function is increasing and those for which the function is decreasing.

a) 

b) 

c) Determine intervals where the function $h(x) = -2x^2 + 4x - 3$ is increasing and those for which the function is decreasing.

Definitions: A graph is **symmetric with repect to the *y*-axis** if the point $(-x, y)$ is on the graph whenever (x, y) is on the graph. A graph is **symmetric with repect to the *x*-axis** if $(x, -y)$ is on the graph whenever (x, y) is on the graph. A graph is **symmetric with repect to the origin** if $(-x, -y)$ is on the graph whenever (x, y) is on the graph.

Definitions: Let *f* be a function. Then, *f* is an **even function** if $f(-x) = f(x)$. The graph of an even function has *y*-axis symmetry. *f* is an **odd function** if $f(-x) = -f(x)$. The graph of an odd function has origin symmetry.

Exercise 2.20: Determine whether each function is even, odd, or neither.

a) $f(x) = x^4 + 3x^2 + 1$

b) $g(x) = x^3 - 3x$

c) $h(x) = \sqrt{1 - x^2}$

d) $F(x) = x^3 + 1$

e) $f(x) = \sqrt{2x^2 + 5}$

f) $g(x) = \dfrac{\sqrt{2x + 5}}{x - 3}$

g) $h(x) = \dfrac{x}{x^2 - 5}$

Answers:

Exercise 2.1

a) $d = \sqrt{181}$

b) $\left(2, -\tfrac{1}{2}\right)$

c) $m = \tfrac{9}{10}$

d) $(x-2)^2 + (y+3)^2 = 45$

Exercise 2.2

a) $y = 5$

b) $x = -2$

c) $y = -\dfrac{5}{3}(x+2) + 5$

d) $y = \dfrac{3}{5}(x+2) + 3$

e) $y = -\dfrac{5}{3}(x+2) + 3$

Exercise 2.3

a) $x = -\tfrac{14}{3}$

b) $x = \tfrac{21}{5}$

c) $x = -2$

d) $t = \tfrac{40}{3}$

e) $x = \tfrac{12}{13}$

f) $x = -\tfrac{9}{16}$

g) $x = -\tfrac{8}{17}$

Exercise 2.4

a) $(-\infty, -1)$

b) $\left(-\infty, \tfrac{17}{12}\right)$

c) $\left(-\infty, -\tfrac{15}{8}\right]$

d) $[19, \infty)$

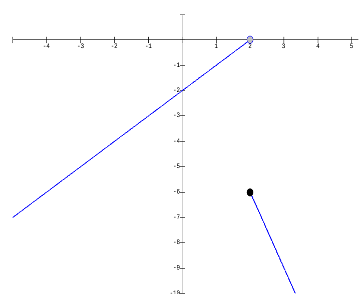
e) $\left(2, \tfrac{46}{5}\right]$

Exercise 2.5

a) Domain is $[-5, 5]$

$f(-3) = -5\,; f(2) = -6\,;\ f(4) = -12\,;\ f(5) = -15$

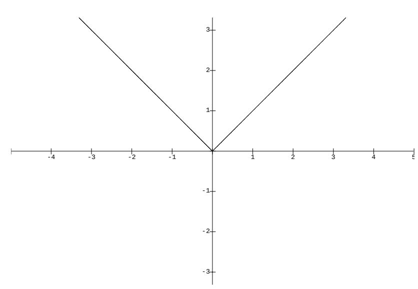
f is discontinuous at $x = 2$



b) Domain is $(-\infty, \infty)$

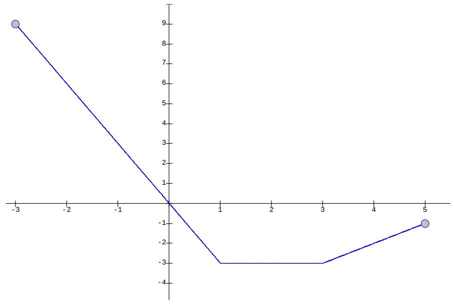
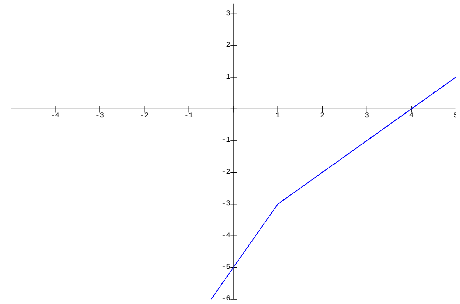
$f(-3) = 3\,; f(2) = 2\,;\ f(4) = 4\,;\ f(5) = 5$

f is continuous



c)

d)

Exercise 2.7

a) $x = \pm 5$ b) $x = -4, 6$ c) $x = -2, 8/3$

Exercise 2.8

a) $(-9, 9)$
b) $(-3, 7)$
c) $[-2, 9]$
d) $(-\infty, -4) \cup (4, \infty)$
e) $(-\infty, -5) \cup (9, \infty)$
f) $(-\infty, -3] \cup [13/3, \infty)$
g) $(-\infty, -1] \cup [5/3, \infty)$

Exercise 2.9

a) $g(x) = -x^2 + 4x - 3$
b) $g(x) = -\frac{1}{2}x^2 + 3x - \frac{13}{2}$
c) $f(x) = (x+2)^2 - 6$; V$(-2, -6)$
d) $F(x) = \left(x + \frac{3}{2}\right)^2 - 2$; V$\left(-3/2, -2\right)$
e) $h(x) = -2(x-1)^2 - 1$; V$(1, -1)$

Exercise 2.10

a) $f(x) = -\frac{2}{3}(x+3)^2 + 5$
b) $f(x) = \frac{3}{8}(x-2)^2 - 3$
c) $f(x) = 3(x-2)^2 + 4$

Exercise 2.11

a) $x = -2, 5$ b) $x = -2, -3$ c) $x = -2, 7$

Exercise 2.12

a) $x = \pm 4$
b) $x = -1, \frac{3}{2}$
c) $x = -1, 4$
d) $x = -\frac{3}{2}, \frac{1}{3}$
e) $x = -\frac{5}{2}, \frac{2}{3}$
f) $x = -\frac{3}{2}, -\frac{2}{3}$

Exercise 2.13

a) $x = \pm 5\sqrt{2}$
b) $x = \pm 1$
c) $x = 5 \pm 2\sqrt{5}$
d) $x = -3 \pm \sqrt{5}$
e) $x = \frac{15 \pm \sqrt{57}}{3}$

Exercise 2.14

a) $x = \frac{9 \pm \sqrt{137}}{2}$
b) $x = 1 \pm \sqrt{5}$
c) $x = -1 \pm \sqrt{5}$
d) $x = \frac{7 \pm \sqrt{17}}{4}$
e) $x = \frac{3}{2}$

Exercise 2.15

a) $x = 1,5$
b) $x = 0,6$
c) $x = \frac{2 \pm \sqrt{19}}{3}$
d) $x = \frac{5 \pm \sqrt{145}}{12}$
e) $x = \frac{3 \pm \sqrt{309}}{30}$

Exercise 2.16

a) $x \approx 22.63''$
b) $x \approx 104$ units
c) $t = 20$ seconds
d) $y \approx 14.05$ ft
e) $500,000$ units
$\$125,000,000$
f) 81.125 ft, 315.47 ft
g) $C(x) = 3.17x + 75000$
$R(x) = 5.25x$
$P(x) = 2.08x - 75000$
h) $t \approx 1.26, 14.86$ seconds
i) In 1987, $x \approx 2.158$

Exercise 2.17

a) $(-1,9)$
b) $[1963,1989]$
c) $f(x) \approx 0.002(x-1950)^2 + 0.5$
$f(1975) \approx \$1.75$
$\approx (2015,2018)$ Answers may vary depending on your function.
d) $r > 8.01\%$
e) (2933,74566)
f) (1.26,14.86)
g) (2579,80752)

Exercise 2.18

a) shifted down 4 units
b) shifted left 4 units, shifted down 4 units, expand by a factor of 4
c) shifted right 4 units
d) *x*-axis reflection, shifted up 4 units, contract by a factor of 1/4
e) *x*-axis reflection, shifted down 4 units, expand by a factor of 4

Exercise 2.19

a) increasing on $(-\infty,-2)$; decreasing on $(-2,\infty)$
b) decreasing on $(-\infty,2)$; increasing on $(2,\infty)$
c) increasing on $(-\infty,1)$; decreasing on $(1,\infty)$

Exercise 2.20

a) even
b) odd
c) even
d) neither
e) even
f) neither
g) odd