

Section 1.7 Function Transformations Answers

Prerequisite Skills Review

- Find $f(-2)$ if $f(x) = -x^2 - 2$. -4
- Make a table of integer values on the interval $-3 \leq x \leq 3$ for the function $f(x) = -x^2$.

x	-3	-2	-1	0	1	2	3
$f(x) = -x^2$	-9	-4	-1	0	-1	-4	-9

- An employee makes wages of \$10.50 an hour. Write a function $W(t)$ that gives total wages for a week if the employee works t hours. $W(t) = 10.50t$

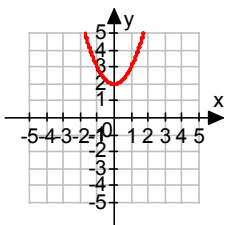
Writing

- Describe a rigid transformation. **when every point of a function is moved in the same way**
- Describe how the graph of $y = f(x - 3)$ is related to the graph of $y = f(x)$. **moved 3 units right**
- Describe how the graph of $y = f(x) + 3$ is related to the graph of $y = f(x)$. **moved 3 units up**
- Describe how the graph of $y = f(x + 2) - 4$ is related to the graph of $y = f(x)$. **moved 2 units left and 4 units down**
- Describe how the graph of $y = -f(x)$ is related to the graph of $y = f(x)$. **reflected vertically across the x-axis**
- Describe how the graph of $y = c f(x)$ is related to the graph of $y = f(x)$, **stretched vertically**
 - if c is a positive constant. **Stretched/compressed vertically by a factor of c**
 - if c is a negative constant. **Stretched/compressed vertically by a factor of c and reflected across the x-axis**
 - if c is zero. **the new graph is the x-axis**

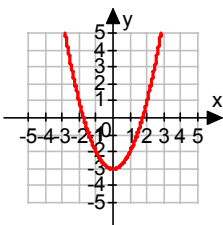
Exercises

- Sketch the resulting graph if the graph of the function shown at right is shifted:

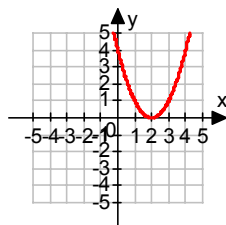
a. up 2 units.



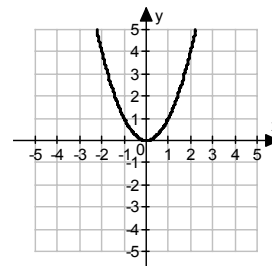
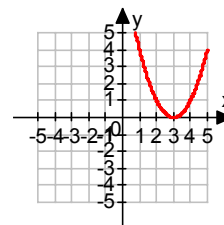
b. down 3 units.



c. left 2 units.



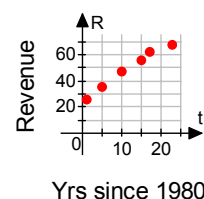
d. right 3 units.



- The following table gives revenue (in billions of dollars) for airline in the U.S. Let the independent variable be the number of years since 1980 and let the dependent variable be revenue. Create a scatterplot of the data.

Year	1981	1985	1990	1995	1997	2003
Revenue (in billions \$)	26	35	47	56	62	67

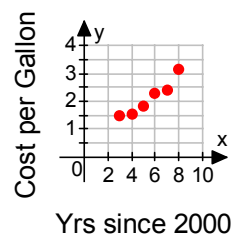
Source: *Bureau of Transportation*



- The following table gives the cost per gallon of regular gasoline in California for the given years. Let the independent variable be years since 2000 and let the dependent variable be the cost per gallon. Create a scatterplot of the data.

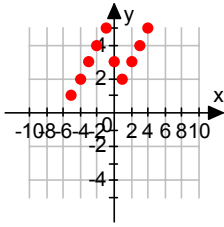
Years	2003	2004	2005	2006	2007	2008
Cost per gallon \$	1.487	1.552	1.824	2.281	2.382	3.159

Source: *Energy Information Administration*

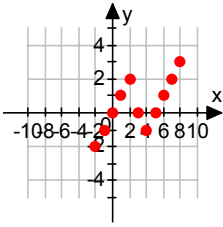


For Exercises 13 – 23, use the graph of f shown to sketch the graph of the translated functions listed below.

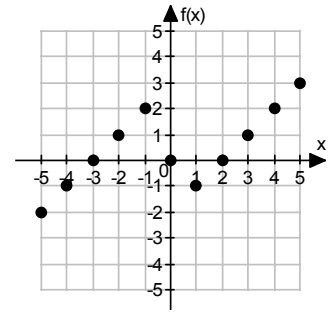
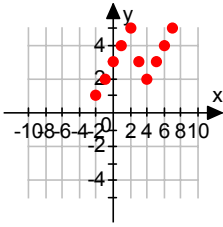
13. $y = f(x) + 3$



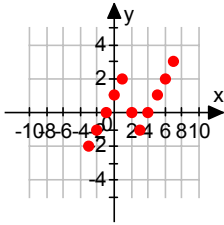
14. $y = f(x - 3)$



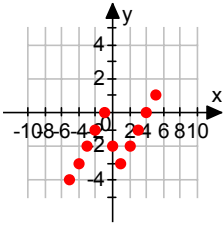
15. $y = f(x - 3) + 3$



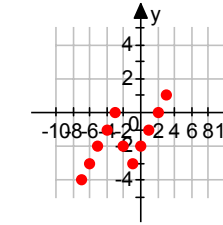
16. $y = f(x - 2)$



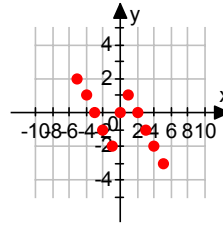
17. $y = f(x) - 2$



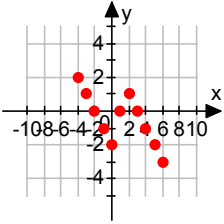
18. $y = f(x - 2) - 2$



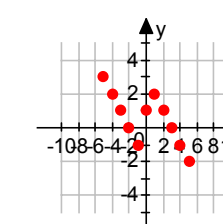
19. $y = -f(x)$



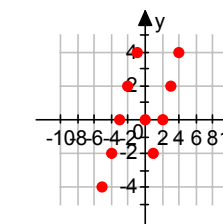
20. $y = -f(x - 1)$



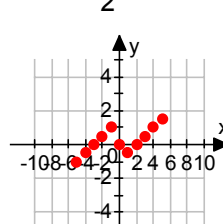
21. $y = -f(x + 1)$



22. $y = 2f(x)$

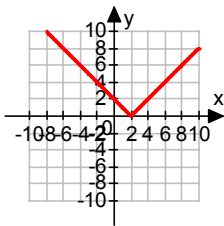


23. $y = \frac{1}{2}f(x)$

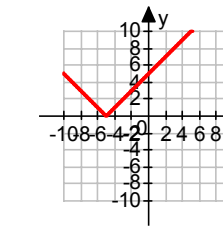


For Exercises 24 – 36, use the graph of f shown to sketch the graph of the translated functions listed below.

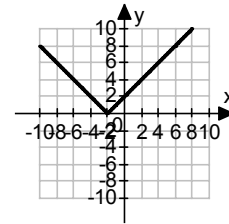
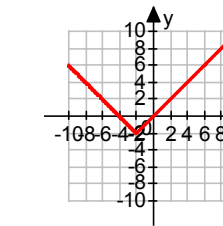
24. $y = f(x - 4)$



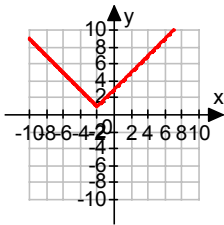
25. $y = f(x + 3)$



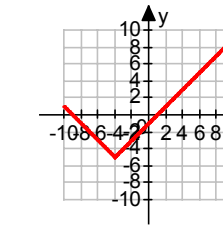
26. $y = f(x) - 2$



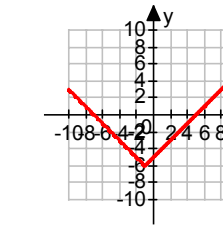
27. $y = f(x) + 1$



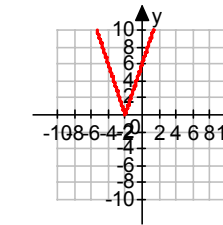
28. $y = f(x + 2) - 5$



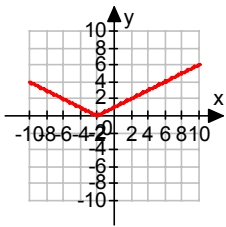
29. $y = f(x - 1) - 6$



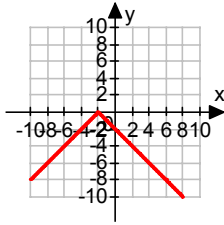
30. $y = 3f(x)$



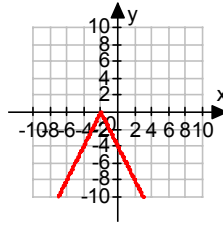
31. $y = \frac{1}{2}f(x)$



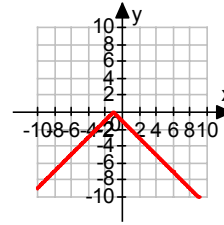
32. $y = -f(x)$



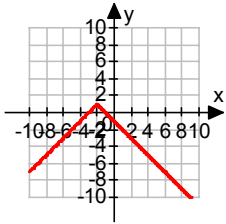
33. $y = -2f(x)$



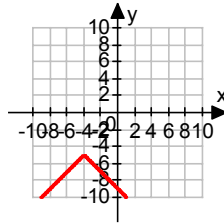
34. $y = -f(x - 1)$



35. $y = -f(x) + 1$

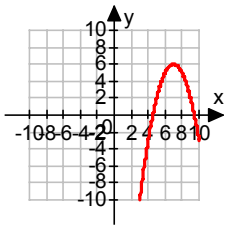


36. $y = -f(x + 2) - 5$

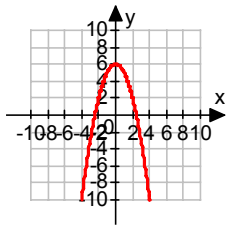


For Exercises 37 – 48, use the graph of f shown to sketch the graph of the translated functions listed below.

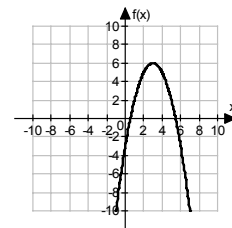
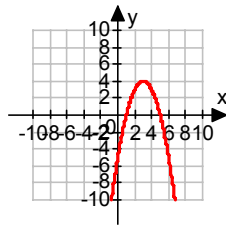
37. $y = f(x - 4)$



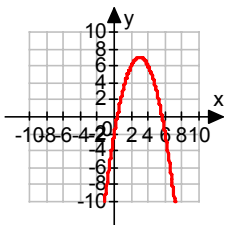
38. $y = f(x + 3)$



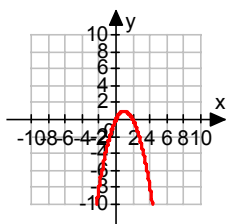
39. $y = f(x) - 2$



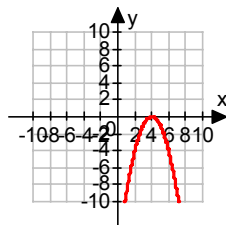
40. $y = f(x) + 1$



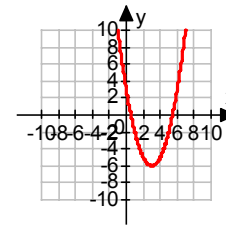
41. $y = f(x + 2) - 5$



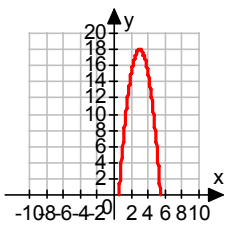
42. $y = f(x - 1) - 6$



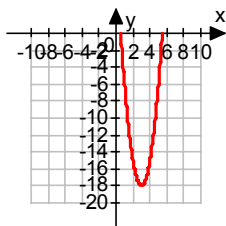
43. $y = -f(x)$



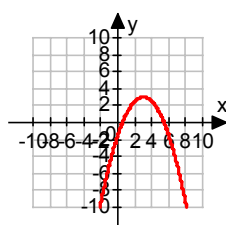
44. $y = 3f(x)$



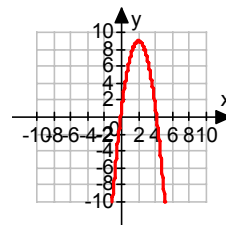
45. $y = -3f(x)$



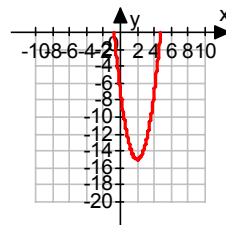
46. $y = \frac{1}{2}f(x)$



47. $y = 2f(x + 1) - 3$



48. $y = -2f(x + 1) - 3$



49. The first table below gives the area of a circle of radius r . Complete the following tables and interpret the meaning of the values of $f(r+2)$ and $f(r) - \pi$.

r	2	3	4.5	8	12	20
$f(r)$	12.57	28.27	63.62	201.06	452.39	1256.60

a.

r	0	1	2.5	6	10	18
$f(r+2)$	12.57	28.27	63.62	201.06	452.39	1256.60

b.

r	2	3	4.5	8	12	20
$f(r) - \pi$	$12.57 - \pi$	$28.27 - \pi$	$63.62 - \pi$	$201.06 - \pi$	$452.39 - \pi$	$1256.60 - \pi$

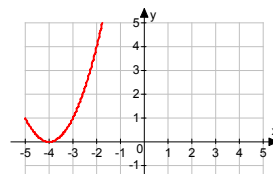
50. Make a table of values and sketch the graph of $f(x) = x^2$ for x -values between -3 and 3 . Then make a table of values and sketch the graphs of the following translated functions.

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9

- a. $g(x) = f(x+4)$. b. $g(x) = f(x)+4$. c. $g(x) = f(x-2)+3$. d. $g(x) = f(x+2) - 3$

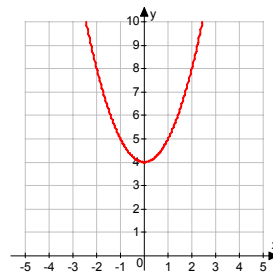
a.

x	-3	-2	-1	0	1	2	3
$x+4$	1	2	3	4	5	6	7
$f(x+4) = (x+4)^2$	1	4	9	16	25	36	49



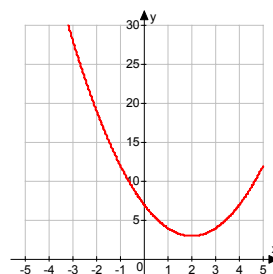
b.

x	-3	-2	-1	0	1	2	3
$f(x) = x^2 + 4$	13	8	5	4	5	8	13



c.

x	-3	-2	-1	0	1	2	3
$x-2$	-5	-4	-3	-2	-1	0	1
$f(x) = (x-2)^2 + 3$	28	19	12	7	4	3	4

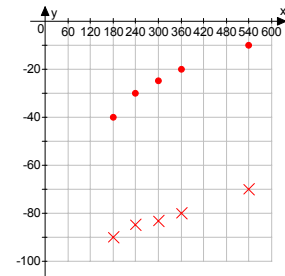


Extending Concepts

51. The pharmaceutical company Proctor and Gamble makes Actonel® and Didronel®, drugs used to treat progressive bone loss. In a clinical study, the mean percent change in serum alkaline phosphates excess (SAPE), a factor that contributes to bone loss, was measured in subjects given Actonel or Didronel. The following table of values gives the mean percent change for subjects given Actonel and Didronel during the follow-up period, which extended from 180 days to 540 days after the treatment began.

Day	180	240	300	360	540
Actonel % Change	-40	-30	-25	-20	-10
Didronel % Change	-90	-85	-83	-80	-70

Source: [Proctor & Gamble Pharmaceuticals](#)



• Actonel x Didronel

- Sketch the graph of both groups on one set of axes. State the domain and range of both sets of data.
Both domains: $\{180, 240, 300, 360, 540\}$
Actonel range: $\{-40, -30, -25, -20, -10\}$ Didronel range: $\{-90, -85, -83, -80, -70\}$
- Let $D(t)$ represent the mean percent change of SAPE of subjects given Didronel t days after the study began. Find the average difference between SAPE. **-56.6% average difference** Write a function in terms of $D(t)$ that approximates the percent change in SAPE of subjects given Actonel. $A(t) = D(t) + 56.6$
- If $D(t) = 0.054t - 98.97$ approximates the change in SAPE of subjects given Didronel, write a function that approximates the change in SAPE given the Actonel.
 $A(t) = 0.054t - 39.37$

52. Chantix™ is a prescription drug used to help people stop smoking. The following table gives the percentage of people given Chantix and a placebo who continuously abstained from smoking t weeks after the study began.

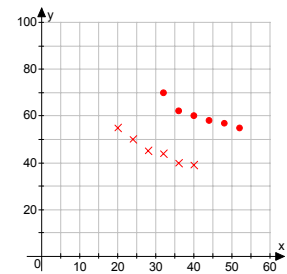
Weeks	32	36	40	44	48	52
Chantix %	70	62	60	58	57	55

Weeks	20	24	28	32	36	40
Placebo %	55	50	45	44	40	39

Source: [Pfizer Labs](#)

Chantix: domain $\{32, 36, 40, 44, 48, 52\}$; range $\{55, 57, 58, 60, 62, 70\}$

Placebo: domain $\{20, 24, 28, 32, 36, 40\}$; range $\{39, 40, 44, 45, 50, 55\}$

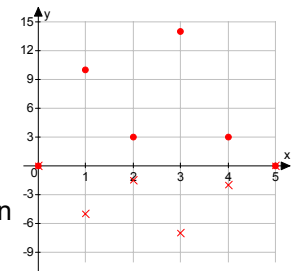


- Sketch the graph of both groups on one set of axes. State the domain and range of each function and describe the relationship between the graphs.
- If $C(t)$ represents the percentage in the Chantix group who had abstained through week t after the study began, write a function in terms of $C(t)$ that gives the percentage of the placebo group who abstained. $P(t) = C(t - 32)$
- If the function $C(t) = 0.038t^2 - 3.845t + 153.1$ is a good model for the Chantix group for $32 \leq t \leq 52$, write a function that models the placebo group. $P(t) = 0.038(t - 32)^2 - 3.845(t - 32) + 153.1$

53. The following table gives the change in systolic blood pressure of healthy volunteers while performing tasks t hours after being given 1 dose of drug SC or a placebo.

t hours after dose	0	1	2	3	4	5
Drug	0	10	3	14	3	0
Placebo	0	-5	-1.5	-7	-2	0

Source: [Pfizer Labs](#)



- Sketch the graph of both groups on one set of axes. State the domain and range of each function and describe the relationship between the graphs.
Both domains: $\{0, 1, 2, 3, 4, 5\}$; Drug range: $\{0, 3, 10, 14\}$; Placebo range: $\{-7, -5, -2, -1.5, 0\}$
- If $S(t)$ represents the change in systolic blood pressure of subjects given the drug t days after the study began, write a function in terms of $S(t)$ that gives the change in systolic blood pressure of subjects given the placebo. $P(t) = -0.5 D(t)$

- c. If $S(t) = 1.5t^5 - 15t^4 + 65t^3 - 118t^2 + 77$ approximates the change in blood pressure of subjects given the placebo, write a function that approximates the change in blood pressure of subjects given the drug. $S(t) = -0.5[1.5t^5 - 15t^4 + 65t^3 - 118t^2 + 77]$

54. Suppose the graph of f passes through the points $(-3, -9)$, $(2, -4)$, and $(1, -1)$. When the graph of f is translated, it passes through the new points $(2, -7)$, $(7, -2)$, and $(6, 1)$. Find the translation of f that produced the new points. The original points are shifted 5 units right and 2 units up.