Name _____ Project 3 Differential Equations

Lanchester's Model of Warfare

1. This is a model of warfare. It is assumed that each nation's army is worn down at a rate proportional to the size of the opposing army. If $x(t)$ and $y(t)$ are the sizes of the opposing armies, then

$$\frac{dx}{dt} = -ay$$
$$\frac{dy}{dt} = -bx \qquad (*)$$

The war is over when one of the armies is destroyed. Naturally, negative values for *x* and *y* are of no interest.

a) Solve (*) by elimination.

b) Find the relationship between *x* and *y* by finding an equation for $\frac{dy}{dx}$ from (*) and solving it.

c) Express the system (*) in matrix form and solve it by the eigenvector method.

d) For system (*), show that if the parameters and initial conditions are such that $\frac{dx}{dt}\Big/x = \frac{dy}{dt}\Big/y$, then $x(t)$ and $y(t)$ both approach 0, and the ratio $y(t)\big/x(t)$ is a constant.

e) What relationship between the initial sizes $x(0)$ and $y(0)$ guarantees that *y* wins (that is $x(t) = 0$ in finite time)?