Autonomous Equations and Stability

Definition: A differential equation of the form $\frac{dy}{dt} = f(y)$ is an autonomous differential equation – the independent variable does not occur in the right-hand side of the equation.

In order to find the equilibrium solutions of a differential equation of this form, we set the right-hand side to zero and solve. For these values of *y* the rate of change would be equal to zero.

Example 1: Finding the equilibrium solutions of an autonomous differential equation

For the equation $\frac{dy}{dt} = ay + by^2,\ a > 0, b > 0,\ -\infty < y_0 < \infty$, find the following:

a) The equilibrium solutions
b) Sketch a graph of $f(y)$ versus *y*.
c) Classify the equilibrium solutions as asymptotically stable or unstable
d) Draw the phase line and several graphs of solutions.

Example 2: Finding the equilibrium solutions of an autonomous differential equation

For the equation $\frac{dy}{dt} = y(1 - y^2)$, $-\infty < y_0 < \infty$, find the following:

a) The equilibrium solutions
b) Sketch a graph of $f(y)$ versus y.
c) Classify the equilibrium solutions as asymptotically stable or unstable
d) Draw the phase line and several graphs of solutions.

Logistic Growth Model
Rate of change is jointly proportional to the current population and the difference between some maximum population M and the current population: $\frac{dP}{dt} = kP(M - P)$
This models the spread of infectious diseases, the growth of a business, and the spread of a rumor. Your books version: $\frac{dy}{dt} = r\left(1 - \frac{y}{K}\right)y$. See the derivation on page 79.

Example 3: Logistic Growth

For the equation $\frac{dy}{dt} = y(1 - y^2)$, $-\infty < y_0 < \infty$, find the following:

a) The equilibrium solutions
b) Sketch a graph of $f(y)$ versus *y*.
c) Classify the equilibrium solutions as asymptotically stable or unstable
d) Draw the phase line.
e) Solve the equation.