Differential Equations

Test 1

Summer 2005

Mike Huff

Instructions: Do 11 of the following 12 problems. You may do the extra credit problem for 5 points extra credit. Extra credit will only be awarded for a complete and correct solution that shows all work.

Name ___________________________ Test 1 Differential Equations 2005 Mike Huff

1. Find values of m so that the function $f(x) = e^{mx}$ is a solution of the differential equation

$$2y'' + 5y' - 3y = 0$$

2. Solve the initial value problem: $yy' = x \sin x e^{-y}$, $y(0) = 1$.

3. Initially there are 50 pounds of salt dissolved in a tank holding 300 gallons of water. A brine solution is pumped into the tank at a rate of 3 gallons per minute, and a well-stirred mixture is then pumped out at the same rate. If the concentration of the solution entering the tank is 2 pounds per gallon, answer the following:

    a) Find a function that gives the amount of salt in the tank at any time.

    b) Determine the amount of salt in the tank after 50 minutes.

    c) How much salt will be in the tank in the long run, that is, as $t \to \infty$?

3. The half-life of a radioactive isotope is the amount of time it takes for a quantity of the material to decay to half the original amount. The half life of Iodine 131 (I-131) is 8 days. Find the following:

a) The decay rate parameter for I-131.

b) This isotope is used in the treatment of hyperthyroid. Suppose it takes 72 hours to from the producer to the hospital. What percent of the original amount shipped actually arrives at the hospital?

c) If the I-131 is stored in the hospital for an additional 48 hours before it is used, how much of the original amount shipped from the producer is left when it is used?

4. Find the general solution of the differential equation $y' = \dfrac{xy + x - y - x^2}{xy - y^2}$.

5. Find a particular solution of the differential equation $y' = (5 - 2y)y$, $y(0) = 1$.

6. Suppose a student carrying a flu virus returns to an isolated college campus of 1000 students. If it assumed that the rate at which the virus spreads is proportional to not only the number x of infected students but also to the number of students not infected, determine the number of infected students after 6 days if it is further observed that after 4 days there are 50 students infected, $x(4) = 50$. Hint: Use the equation $\frac{dx}{dt} = kx(1000 - x)$.

7. Find the general solution of the differential equation $(t^2 + 1)\frac{dy}{dt} = 2ty + t^3$

8. Find the general solution of the differential equation
$\left(e^{2y} - y\cos(xy)\right)dx + \left(2xe^{2y} - x\cos(xy) + 2y\right)dy = 0$

9. Find the general solution of the differential equation
$\left(1 + 2xye^{y}\right)dx + x\left(xe^{y} - 1\right)dy = 0$

10. The modified logistic equation is given by $\frac{dP}{dt} = k\left(1 - \frac{P}{N}\right)\left(\frac{P}{M} - 1\right)P$. Find the following:

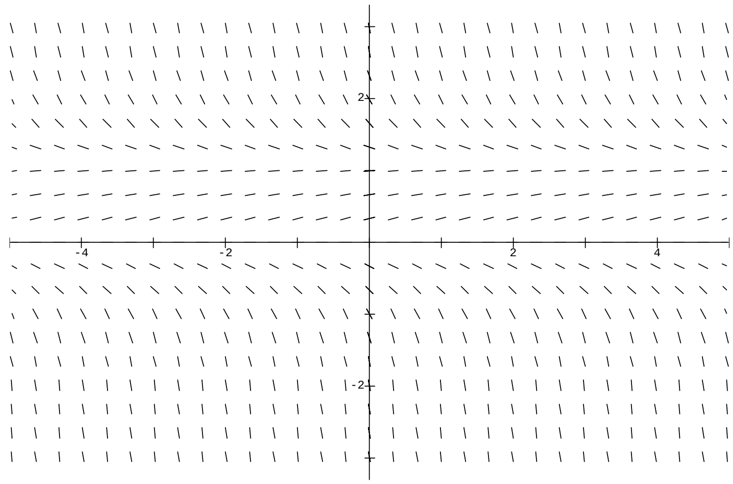
a) The equilibrium solutions

b) Sketch a graph of $f(P)$ versus *P*.

c) Classify the equilibrium solutions as asymptotically stable or unstable

d) Draw the phase line and several graphs of solutions.

12. Sketch the solution curves on the following vector field that correspond to the initial conditions $y(0) = 2$, $y(0) = -1$, and $y(0) = 0$. How do the three solutions differ?



The direction field drawn above corresponds to the differential equation $y' = (1 - y)y$.

Find the general solution of this equation. Find a particular solution satisfying the initial condition $y(0) = 2$.

Extra Credit: Solve the Bernoulli equation $\frac{dy}{dx} = ay + by^2$