Differential Equations

Test 1

Summer 2005

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Instructions: Do 11 of the following 12 problems. You may do the extra credit problem for 5 points extra credit. Extra credit will only be awarded for a complete and correct solution that shows all work.

Name ___________________________ Test 1 Differential Equations 2005 Mike Huff

1. Find values of m so that the function $f(x) = x^m$ is a solution of the differential equation

$$x^2 y'' - y = 0$$

2. Solve the differential equation: $\dfrac{dy}{dx} + (\tan x)\, y = 0$.

3. Initially there are 25 pounds of salt dissolved in a tank holding 200 gallons of water. A brine solution is pumped into the tank at a rate of 4 gallons per minute, and a well-stirred mixture is then pumped out at the same rate. If the concentration of the solution entering the tank is 2 pounds per gallon, answer the following:

   a) Find a function that gives the amount of salt in the tank at any time.

   b) Determine the amount of salt in the tank after 15 minutes.

   c) How much salt will be in the tank in the long run, that is, as $t \to \infty$?

4. The half-life of a radioactive isotope is the amount of time it takes for a quantity of the material to decay to half the original amount. The half life of Carbon 14 (C-14) is 5320 years. Find the following:

a) The decay rate parameter for C-14.

b) Carbon dating is a method that can be used to determine the time that has elapsed since the death of organic matter. The assumptions are that C-14 makes up a constant proportion of the carbon that living matter ingests on a regular basis and that once matter dies, the C-14 present decays. Find the time since death if 88% of the original C-14 is still present.

c) Find the time since death if 2% of the original C-14 is still present.

5. Find the general solution of the differential equation $y' = \dfrac{xy + 2y - x - 2}{xy - 3y + x - 3}$.

6. Find a particular solution of the differential equation $y' = (4 - 3y)y$, $y(0) = 1$.

7. Suppose a student carrying a flu virus returns to an isolated college campus of 5,000 students. If it assumed that the rate at which the virus spreads is proportional to not only the number x of infected students but also to the number of students not infected, determine the number of infected students after 10 days if it is further observed that after 3 days there are 100 students infected, $x(3) = 100$. Hint: Use the equation $\frac{dx}{dt} = kx(1000 - x)$.

8. Find the general solution of the differential equation $t\frac{dy}{dt} - 4y = t^6 e^t$.

9. Solve the initial value problem $\left(\cos(x)\sin(x) - xy^2\right)dx + y\left(1 - x^2\right)dy = 0$, $y(0) = 2$

10. Find the general solution of the differential equation

$$\left(4xy + 3y^2 - x\right)dx + x\left(x + 2y\right)dy = 0$$

11. For the differential equation $\frac{dy}{dt} = k(1+y)(y-1)y$, $k > 0$. Find the following:

a) The equilibrium solutions

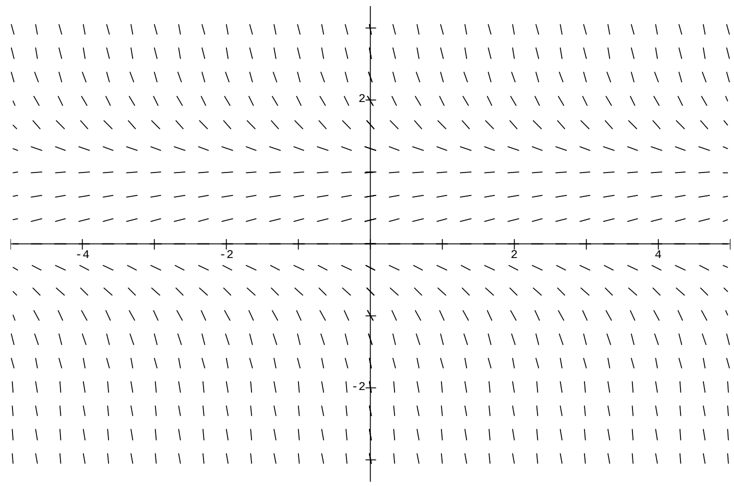
b) Sketch a graph of $f(y)$ versus y.

c) Classify the equilibrium solutions as asymptotically stable or unstable

d) Draw the phase line and several graphs of solutions.

e) What is $\lim_{t \to \infty} y(t)$ for the solution that satisfies $y(0) = \frac{1}{2}$?

12. Sketch the solution curves on the following vector field that correspond to the initial conditions $y(-2) = 2$, $y(-2) = -2$, and $y(-2) = 0$. How do the three solutions differ?



The direction field drawn above corresponds to the differential equation $y' = (1 - y)\,y$.

Find the general solution of this equation. Find a particular solution satisfying the initial condition $y(0) = -2$.

Extra Credit: Solve the Bernoulli equation $\frac{dy}{dx} = ay + by^2$