

Name _____ Test 1 Differential Equations 2002 Mike Huff

1. Verify that the function $f(x) = x^2 \ln x$ is a solution of the differential equation

$$x^2 y'' - 3xy' + 4y = 0$$

2. A ball with a mass of 0.25 kg is thrown upward with an initial velocity of 20 m/sec from the roof of a building 30 m high. Neglect air resistance.
- Find the maximum height above the ground the ball reaches.
 - Assuming the ball misses the building on the way down, find the time it takes for the ball to hit the ground.

3. Solve the initial value problem: $y' = xe^{-x}$, $y(0) = 1$.

4. Find the position function $x(t)$ of a moving particle with the acceleration function $a(t) = \frac{1}{(t+1)^3}$, initial position $x(0) = 0$ and initial velocity $v(0) = 0$.

5. Find the general solution of the differential equation $y' = \frac{xy + x - y - x^2}{xy - y^2}$.

6. Find the general solution of the differential equation $(x^2 + 1)y' = 2xy + x^3$.

7. Find a particular solution of the differential equation $y' = (5 - 2y)y$, $y(0) = 1$.

8. A pitcher of iced tea initially $25^\circ C$ is to be cooled by placing it on the front porch, where the temperature is $0^\circ C$. Suppose that the temperature of the iced tea has dropped to $15^\circ C$ after 20 min. When will it be at $5^\circ C$?

9. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{(x-1)y^5}{x^2(2y^3-y)}$.

10. Write a differential equation that is a mathematical model of the situation described:

a) The time rate of change of population P is proportional to the square root of P .

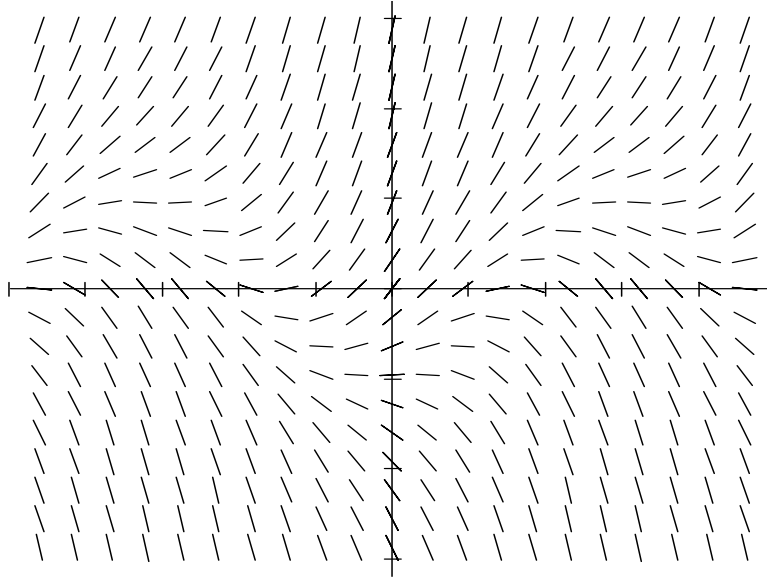
b) The time rate of change of the velocity v of a coasting motorboat is proportional to the square of v .

c) In a city having a fixed population P of persons, the time rate of change of the number N of those persons who have heard a certain rumor is proportional to the number of those who have not heard the rumor.

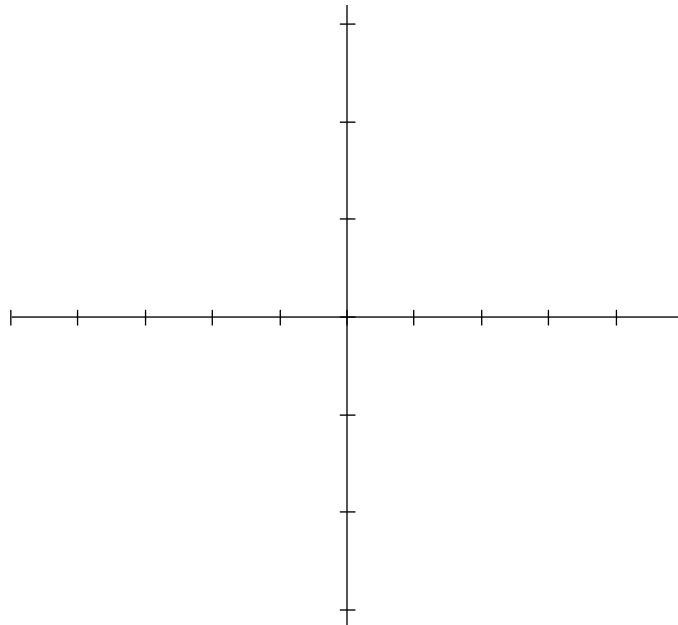
d) Solve one of the equations you have written above.

11.

- a) Sketch the solution curves on the following vector field that correspond to the initial conditions $y(0) = 1$ and $y(0) = -1$. How do the two solutions differ?



- b) Describe the isoclines of the differential equation $\frac{dy}{dx} = x - y^2$. Draw a sketch showing three of the isoclines, each marked with short line segments having the appropriate slope.



12. The direction field drawn in problem 11 a) corresponds to the differential equation $y' = \cos x + y$. Find the general solution of this equation. Find particular solutions satisfying the initial conditions $y(0) = 1$ and $y(0) = -1$.

Differential Equations

Test 1

Summer 2002

Mike Huff

Instructions: Do 10 of the following 12 problems. You may do one problem for 5 points extra credit. Extra credit will only be awarded for a complete and correct solution.