

If the function f and g are continuous on an open interval I containing the point t_0 and that $F(t)$ is any antiderivative of f in I , then the unique solution $y(t)$ of the IVP $y' + f(t)y = g(t)$, $y(t_0) = y_0$ lives on I . It has an interpretation in terms of initial data y_0 and input $g(t)$:

$$y(t) = \text{Response to initial data} + \text{Response to input}$$

$$y(t) = \left[e^{F(t_0)} y_0 e^{-F(t)} \right] + \left[e^{-F(t)} \int_{t_0}^t e^{F(s)} g(s) ds \right]$$

The **response to initial data** $y(t) = \left[e^{F(t_0)} y_0 e^{-F(t)} \right]$ solves the *undriven IVP*:

$$y' + f(t)y = 0, \quad y(t_0) = y_0.$$

The **response to input** $y(t) = e^{-F(t)} \int_{t_0}^t e^{F(s)} g(s) ds$ solves the driven IVP:

$$y' + f(t)y = g(t), \quad y(t_0) = 0.$$

Structure of the General Solution of a Linear ODE

If the function f and g are continuous on an interval I . Then the general solution $y(t)$ of the driven linear ODE $y' + f(t)y = g(t)$ is given by

$$y(t) = y_g(t) + y_p(t), \quad \text{all } t \text{ in } I$$

where $y_g(t)$ is the general solution of the *undriven* linear ODE $y' + f(t)y = 0$ and $y_p(t)$ is a particular solution of the *driven* linear ODE $y' + f(t)y = g(t)$.

This explains why the guessing method (**method of undetermined coefficients**) works if the driving term is nice enough.

Example: Solve using the method of undetermined coefficients

$$y' + 0.1y = 10(0.2 + 0.1\sin(t)); y(0) = 15$$

