Separable Differential Equations

Definition: A differential equation that can be written in the form $g(y)\frac{dy}{dt} = f(t)$ is called a **separable differential equation**.

Using differentials we can write this equation in the form $g(y)dy = f(t)dt$ and then integrate both sides to get $\int g(y)dy = \int f(t)dt$. If $G(y)$ is any antiderivative of $g(y)$ and $F(t)$ is any antiderivative of $f(t)$, then the equation $G(y) = F(t) + C$ defines a family of solutions of the original equation implicitly. Under certain conditions we will be able to solve for the function $y(t)$ explicitly.

Example 1: Solving a separable differential equation

Find the general solution of the equation $\frac{dy}{dt} = \frac{t^2}{y^2}$

Example 2: Solving a separable differential equation

Find the general solution of the equation $e^y \dfrac{dy}{dt} - t - t^3 = 0$

Example 3: Solving an initial-value problem

Solve the initial-value problem $e^y \dfrac{dy}{dt} - t - t^3 = 0\,,\; y(1) = 1$

Example 4: Solving an initial-value problem

Solve the initial-value problem $(4y - \cos y)\dfrac{dy}{dx} - 3x^2 = 0,\ y(0) = 0$

Example 5: Solving an initial-value problem

$$y' = \frac{2x}{y + x^2 y}, \quad y(0) = -2$$

a) Solve the initial value problem in explicit form

b) Plot the graph of the solution

c) Determine the interval in which the solution is defined.