Section 3 Solving Linear Equations

Definitions: An **equation** is a statement about the relationship between two algebraic expressions. The truth of an equation may depend upon the value of the variable or variables. Such statements are called **conditional statements**. Some equations are true (or false) regardless of the value assigned to the variable. These are called **identities**.

Example 1: Example of equations and conditional statements
The following are examples of equations.

a) $2x^2 + 5x + 3 = 0$
b) $\sqrt{2}x = 3$
c) $|2x - 3| = 7$
d) $2x + 5(x + 3) = 0$

The equation $x + 15 = 18$ is neither true nor false (it's a conditional statement), since if $x = 3$ it is true but if $x = 10$ it is false.

Definitions: A **linear equation** in one variable is an equation that can be written in the form $ax + b = 0$. Here *a* and *b* are real numbers with $a \neq 0$. **Solving** a linear equation means finding the value(s) of the variable that make the equation true. The **solution** is the set of values of the variable that make the equation true.

Example 2: $x = 2$ is a solution of the equation $x + 5 = 7$ because, when 2 is substituted in for *x* the result is a true statement: $2 + 5 = 7$ is true.

The Addition Property of Equality: If $a = b$ then $a + c = b + c$. This says that you can add the same number to both sides of an equation without changing the solution.

Example 3: Solving Equations with the Addition Property

Solve the equation $x - 5 = 7$.

Solution: If $x - 5 = 7$ then we can add 5 to both sides of the equation:

$$\begin{aligned} x - 5 &= 7 \\ x - 5 + 5 &= 7 + 5 \quad \text{Addition property} \\ x + 0 &= 12 \\ x &= 12 \end{aligned}$$

Example 4: Solving Equations with the Addition Property

Solve the equation $-\frac{5}{3}+x=\frac{7}{4}$

Solution: If $-\frac{5}{3}+x=\frac{7}{4}$ then we can add $\frac{5}{3}$ to both sides of the equation:

$$-\frac{5}{3}+x=\frac{7}{4}$$

$$x=\frac{7}{4}+\frac{5}{3}=\frac{21}{12}+\frac{20}{12}=\frac{41}{12}$$

The Multiplication Property of Equality: If $a=b$ then $ac=bc$. This says that you can multiply both sides of an equation by the same number without changing the solution.

Example 5: Solving Equations with the Multiplication Property

Solve the equation $\frac{x}{3}=4$.

Solution:If $\frac{x}{3}=4$ then we can multiply both sides of the equation by 3:

$$\frac{x}{3}=4$$

$$3\left(\frac{x}{3}\right)=3\cdot 4$$

$$1\cdot x=12$$

$$x=12$$

Exercises:

a) Why do we not need a Subtraction Property of Equality?

b) Why do we not need a Division Property of Equality?

Definition: Two equations are called **equivalent equations** if they have the same solution.

Important Fact: The two properties we have introduced (Addition Property of Equality, Multiplication Property of Equality) both produce equivalent equations.

Example 6: Solving Equations Using Both Properties
Solve the equation $7 - x = 18$.

Solution:

$$7 - x = 18$$
$$-7 + 7 - x = 18 + (-7) \quad \text{Addition property of equality.}$$
$$0 - x = 11 \quad \text{Additive inverse property.}$$
$$-x = 11 \quad \text{Addition property of zero.}$$
$$\frac{-x}{-1} = \frac{11}{-1} \quad \text{Multiplication property of equality.}$$
$$x = -11$$

Example 7: Solving Equations Using Both Properties
Solve the equation $7x + 3 = 3x - 17$.

Example 8: Solving Equations Using Both Properties
Solve the equation $1.7x + 3.2 = 3.6x - 1.7 - 2.5x$.

Solving Equations Containing Grouping Symbols

To solve a linear equation in one variable:

1. Simplify to terms, that is, remove grouping symbols.
2. Combine like terms on each side of the equation
   Note: You may need to repeat the first two steps several times to remove *all* the grouping symbols.
3. Move any terms that contain the variable to one side by adding inverses to both sides of the equation. (Addition Property of Equality)
4. Isolate the variable:
   a. First, move all constants being added to or subtracted from the variable term to the other side. (Addition Property of Equality)
   b. Finally, complete the solution by multiplying both sides of the equation by the reciprocal of the coefficient of the variable. (Multiplication Property of Equality)
5. Check the solution – Replace the variable with the value you found and simplify both sides of the equation. If the resulting number on each side of the equation is the same, then the solution is correct.

Example 9: Solving Equation Containing Grouping Symbols.

Solve: $0 = 2(x+3) + 3(x-7)$

Example 10: **Solving Linear Equations in One Variable**

Solve the following equations.

a) $8(x+2) - 3(2x+1) = 2(x+5)$

b) $\frac{t}{5} - \frac{t}{2} = 3$

c) $\frac{3}{5}x - \frac{1}{2} = \frac{3}{7}x$

d) $\frac{3}{4}(3x - 6) = 9$

e) $\frac{3}{4}\left(3t - \frac{1}{2}\right) - \frac{2}{3} = \frac{1}{3}$

Appendix: The Properties of Equality

Properties of Equality

1. **Reflexive Property of Equality**: For all real numbers a, $a = a$. That is, any number is equal to itself.

   Example:
   a) $3 = 3$
   b) 3 + y = 3 + y
   c) 3(x + y) = 3(x + y)

2. **Symmetric Property of Equality**: For all real numbers a and b, if a = b, then b = a. That is, we can interchange the two sides of an equation.
   Example:
   a) If 3x + 5 = 7, then 7 = 3x + 5.
   b) If 3(x + y) = z + 5y, then z + 5y = 3(x + y).

3. **Transitive Property of Equality**: For any three whole numbers a, b and c, if a = b and b = c, then a = c.

Example: **Reflexive, Symmetric, and Transitive Properties**

a) If $5 + 12 = 17$ and $17 = 34/2$ then $5 + 12 = 34/2$.
b) If $0 = 8y - 6y$ and $8y - 6y = 2y$ then $0 = 2y$.
c) If $8x - 6 = 12$ and $z = 8x - 6$ then $z = 12$.