

## Section 7 Solving Inequalities

**Definition:** An **inequality** is a statement about the relationship between two expressions. These symbols are used to represent the idea that there is a relationship between the expressions but that the two expressions are *not* identical. We will use the following symbols to represent inequalities.

$a > b$	$a$ is greater than $b$
$a < b$	$a$ is less than $b$
$a \geq b$	$a$ is greater than <i>or equal to</i> $b$
$a \leq b$	$a$ is less than <i>or equal to</i> $b$
$a \neq b$	$a$ is not equal to $b$

**Definition:** A **solution** of an inequality is the set of numbers that makes the inequality true.

### Example 1: Verifying a solution

Is the given number a solution of the inequality  $x < -2$ ?

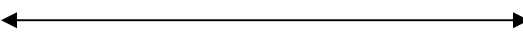
a) -1

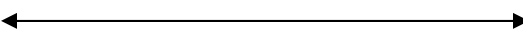
b) -2

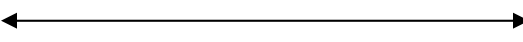
### Graphs of Inequalities

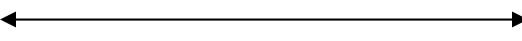
#### Example 2: Graphing Inequalities


Sketch the graphs of the following inequalities


a)  $x > 4$  

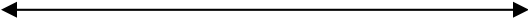
b)  $x \geq 4$  

c)  $x < 4$  

d)  $x \leq 4$  

e)  $-1 < x < 4$   A horizontal number line with arrows at both ends. There are open circles at -1 and 4, and a line segment connects them.

f)  $-1 \leq x < 4$   A horizontal number line with arrows at both ends. There is a closed circle at -1 and an open circle at 4, and a line segment connects them.

g)  $-1 \leq x \leq 4$   A horizontal number line with arrows at both ends. There are closed circles at -1 and 4, and a line segment connects them.

## Solving Inequalities

### The Properties of Inequalities

Each of these rules is still true when  $<$  is replaced by  $\leq$ ,  $>$ , or  $\geq$ . Let  $a$ ,  $b$ , and  $c$  be real numbers. Then, the following properties hold:

**The Addition Property of Inequality:** If  $a < b$ , then  $a + c < b + c$ .

**The Multiplication Properties of Inequality:** If  $a < b$  and  $c > 0$ , then  $a \cdot c < b \cdot c$ .

If  $a < b$  and  $c < 0$ , then  $a \cdot c > b \cdot c$ .

### Example 3 Solving Inequalities

Solve  $x + 5 > 7$ .

### Example 4: Solving Inequalities

Solve  $5x > 7$ .

**Example 5: Solving Inequalities**

Solve  $-x + 5 > 7$ .

**Example 6: Solving Inequalities**

Solve  $-5x > 7$ .

**Example 7: Solving Inequalities**

Solve  $3x - 2 \leq 7x + 4$ .

**Example 8: Solving Inequalities**

Solve  $15.2 - 3.1x \geq -4.7$ .

**Example 9: Solving Inequalities**

Solve  $-5(x - 2) + 1 < 2(x - 1) - 2$ .

**Example 10: Solving Inequalities**

Solve  $\frac{x}{5} + \frac{3}{7} \leq 2$ .

**Example 11: Solving Inequalities**

Solve  $\frac{3}{2} \left( \frac{5}{6} - 4x \right) - \frac{2}{3} \leq \frac{3}{12}$ .