

Polynomials

Types of Polynomials

Definition: A term of the form ax^n where n is a whole number is a **monomial**. A **polynomial** is a monomial or a sum of monomials.

The following are all polynomials:

- a) $127x^5$ is a monomial
- b) $x^2 - 3x$ is a binomial
- c) $x^2 - 3x + 5$ is a trinomial

Example 1: Identifying the Terms of a Polynomial

Find the terms of the following polynomials.

a) $x^2 - 3x + 5$

b) $x^3 + 3x^2 - 2x - 5$

Degree and Coefficient

Definition: The **degree** of a term is the number of variable factors in that term. If there is only one variable, then the degree is the same as the exponent on the variable. The **coefficient** of a term is the constant part of the term.

Example 2: Identifying the Degree of a Term

Find the degree and coefficient of each term

a) $127x^5$

b) $-3x^2$

c) -3

Definition: The **degree** of a polynomial is the same as the degree of its term of highest degree. The **leading coefficient** is the coefficient of the highest degree term.

Example 3: Identifying the Degree and Leading Coefficient of a Polynomial

Find the degree and coefficient of each term

a) $2x^4 - 3x + 5$

b) $-2x^3 - 3x^3 + 5$

c) $12x^4 - 2x^3 - 3x^3 + 5$

Evaluating Polynomials

Polynomials can be evaluated by substituting a numeric value into the expression in place of the variable.

Example 4: Evaluating Polynomials

Evaluate $-x^2 + 4x - 5$ for $x = -2$.

Addition and Subtraction of Polynomials

Example 5: Addition of Polynomials

a) Add: $4x - 5$ and $3x - 15$

b) Add: $(-x^2 + 4x - 5) + (-4x^2 + 3x - 15)$

c) $(x^2 - 3x + 1) + (-3x^2 + 2x - 5)$

Subtraction of Polynomials

The Opposite of a Polynomial

The opposite of $-4x^2 + 3x - 15$ is $-(-4x^2 + 3x - 15) = 4x^2 - 3x + 15$

Example 6: Subtraction of Polynomials

a) Subtract $4x - 5$ from $3x - 15$

b) Subtract $-x^2 + 4x - 5$ from $-4x^2 + 3x - 15$

c) $(x^2 - 3x + 1) - (-3x^2 + 2x - 5)$

d) $x^3 + 3x^2 - 2x - 5 - (-3x^3 - 2x^2 - 3x + 7)$

Multiplication of Polynomials

Multiplying Monomials

Example 7: Multiplying of Monomials

Multiply:

a) $(-4x^2)(-5x^5)$

b) $(4xy^2)(-2x^4y^2)$

Example 8: Multiplying a Monomial times a Polynomial

Multiply:

a) $x(x - 5)$

b) $-x(x^2 - 5x)$

c) $-3x(-2x^2 + 3x - 4)$

Example 9: Multiplying a Monomial times a Polynomial

Multiply:

a) $(x + 2)(x - 5)$

b) $(x + 1)(x - 1)$

c) $(2x - 1)(x - 2)$

d) $(3x^2 - 2)(x^2 - x)$

e) $(x + 1)(x^2 - x - 3)$

f) $(2x - 3)(3x^2 - 2x + 1)$