

# Polynomials in Several Variables

## Degree and Coefficient

**Definition:** The **degree** of a term is the number of variable factors in that term. The **coefficient** of a term is the constant part of the term. The **degree of a polynomial** is the same as the degree of its term of highest degree. The **leading coefficient** is the coefficient of the highest degree term.

The following are all polynomials in several variables:

- a)  $127x^5y^2z$  is a 8<sup>th</sup> degree monomial
- b)  $-12x^2y^2z^5 - 3xy^2z^4$  is a 9<sup>th</sup> degree binomial with leading coefficient -12
- c)  $17x^4y^2z^5 - 3xy^2z^4 + 5xyz$  is an 11<sup>th</sup> degree trinomial with leading coefficient 17

### Example 1: Identifying the Terms of a Polynomial

Find the terms of the following polynomials. Identify the degree of each term and its coefficient.

a)  $127x^5y^2$

b)  $-12x^2y^2z^5 - 3xy^2z^4$

c)  $17x^4y^2z^5 - 3xy^2z^4 + 5xyz$

## Evaluating Polynomials

Polynomials can be evaluated by substituting a numeric value into the expression in place of the variable.

### Example 2: Evaluating Polynomials

Evaluate  $-12x^2y^2 - 3xy^2$  for  $x = -2$  and  $y = -3$

## Addition and Subtraction of Polynomials

### Example 3: Addition of Polynomials

a) Add:  $(4x - 5y) + (3x - 15y)$

b) Add:  $(-x^2 + 4xy - 5y^2) + (-4x^2 + 3xy - 15y^2)$

c) Add:  $(-11x^2y - 4xy^2 - 3y^3) + (-3x^2y - 7xy^2 - 4y^3)$

## Subtraction of Polynomials

### Example 4: Subtraction of Polynomials

a) Subtract  $(4x - 5y) - (3xy - 15y)$

b) Subtract  $-x^2y^3 + 4xy^2 - 5xy$  from  $-4x^2y^3 + 3xy - 15$

## Multiplication of Polynomials

### Multiplying Monomials

#### Example 5: Multiplying of Monomials

Multiply:  $(-4x^2y^3)(-5x^5y^2)$

#### Example 6: Multiplying a Monomial times a Polynomial

Multiply:  $-2x^2y(x^3y^2 - 5xy)$

#### Example 7: Multiplying Polynomials

Multiply:

a)  $(x + 2y)(x - 5y)$

b)  $(x + y)(x - y)$

c)  $(2x - y)^2$

d)  $(3x^2 - 2y^2)(x^2y - xy^2)$

e)  $(x + y^2)^2$

f)  $(2x^3y^2 - 3xy)(3x^2y - 2xy^2 + y^3)$