

Section 15 Simplifying Rational Expressions

Definition: A rational expression is an expression of the form $\frac{p}{q}$, where p and q are polynomials. We must exclude the possibility that $q = 0$ since the expression does not have a value in that case. If q contains a variable then the values of the variable that make the denominator 0 are called **excluded values**.

To find the excluded values, we set the denominator equal to zero and solve. The expression is defined for every value except the excluded ones.

Example 1: Finding excluded values

Determine the value or values for which the expression is defined.

a) $\frac{x - 4}{5x - 2}$

Set $5x - 2 = 0$. Solving, we have, $x = \frac{2}{5}$. So the excluded value is $x = \frac{2}{5}$. The expression is defined for all x except $x \neq \frac{2}{5}$.

b) $\frac{x + 2}{x^2 - 4}$

Answer: $x \neq 2, -2$

c) $\frac{x^2 + 2}{x^2 - 4x + 3}$

Answer: $x \neq 1, 3$

To simplify rational expressions, we factor the numerator and denominator and cancel common factors.

Example 2: Simplify rational expressions.

a) $\frac{6x^4 + 3x^3 + 4x^2}{2x^2}$

b) $\frac{x^2 - 6x + 9}{x^2 - 9}$

c) $\frac{4x^2 - 12x - 40}{2x^2 - 16x + 30}$

d) $\frac{5x - 3}{3 - 5x}$

e) $\frac{2x^2 + 5x - 3}{1 - 2x}$