Section 18 Adding and Subtracting Rational Expressions

Adding Rational Expressions with a Common Denominator

To add or subtract rational expressions that already have a common denominator, write the numerators over the common denominator with the appropriate sign (+ or -) between the numerators. Perform the operation and then simplify the resulting fraction.

Example 1: Adding and Subtracting Rational Expressions

a) $\frac{2}{5}+\frac{11}{5}=\frac{2+11}{5}=\frac{13}{5}$

b) $\frac{11}{15}-\frac{8}{15}=\frac{11-8}{15}=\frac{3}{15}=\frac{1\cdot 3}{5\cdot 3}=\frac{1\cdot \cancel{3}^{1}}{5\cdot \cancel{3}^{1}}=\frac{1}{5}$

c) $\frac{x}{x+2}+\frac{4x-1}{x+2}=\frac{x+4x-1}{x+2}=\frac{5x-1}{x+2}$

d) $\frac{7x}{x-4}-\frac{28}{x-4}=\frac{7x-28}{x-4}=\frac{7(x-4)}{x-4}=\frac{7\,\cancel{(x-4)}^{1}}{\cancel{x-4}^{1}}=7$

e) $\dfrac{x}{x^2+2x-15} - \dfrac{3}{x^2+2x-15}$ **answer:** $\dfrac{1}{x+5}$

f) $\dfrac{2x+3}{x^2-x-30} - \dfrac{x-2}{x^2-x-30}$ **answer:** $\dfrac{1}{x-6}$

g) $\dfrac{3x-1}{x^2+5x-6} - \dfrac{2x-7}{x^2+5x-6}$ **answer:** $\dfrac{1}{x-1}$

Finding the LCD of Rational Expressions

In order to add rational expression with different denominators, we must first find a common denominator (LCD) of the expressions.

Definition: A **multiple** of a given number is a number that the given number will divide into evenly. A **common multiple** of two or more numbers is a number that all the given numbers will divide into evenly. The **least common multiple (LCM)** of a set of numbers is the smallest of the common multiples.

Example: Finding the LCM.

Find the LCM of 6 and 8.

Solution: First, list the multiples of each number.

Multiples of 6 are: $6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, \ldots$
Multiples of 8 are: $8, 16, 24, 32, 40, 48, 56, 64, 72, 80, \ldots$

Next, we identify common multiples.

Multiples of 6 are: $6, 12, 18, \underline{24}, 30, 36, 42, \underline{48}, 54, 60, 66, \underline{72}, 78, \ldots$
Multiples of 8 are: $8, 16, \underline{24}, 32, 40, \underline{48}, 56, 64, \underline{72}, 80, \ldots$

So the common multiples of 6 and 8 are $24, 48, 72, \ldots$. It follows that the least common multiple (LCM) of 6 and 8 is 24.

To add two (or subtract) rational expressions, we first find the least common multiple of the denominators (LCD) and then convert the fractions to equivalent ones with the LCD as the common denominator and then add (or subtract) as before. Finally, we have to simplify the resulting expression.

Example: Finding the LCD of Rational Expressions.

a) $\frac{5}{8}, \frac{7}{12}, \frac{11}{40}$

The LCM of 8, 12, and 40 is 120. Therefore, the LCD of the rational expressions is 120.

b) $\frac{3x+2y}{3y}, \frac{x+2y}{6x}$ The denominators are $3y$ and $6x$. The LCM $3y$ and $6x$ is $6xy$.

Therefore, the LCD of the rational expressions is $6xy$.

Definition: **Equivalent fractions** or rational expressions are just different ways of writing the same number.

Example: The following fractions are equivalent: $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \cdots$

Example: Converting to Equivalent Fractions.

Rewrite each rational expression as an equivalent one with the given denominator.

a) $\frac{7}{5x} = \frac{\quad}{35x^2}$

b) $\frac{11}{x-3} = \frac{\quad}{5(x-3)}$

c) $\frac{x}{x^2+6x+8} = \frac{\quad}{(x+4)(x+2)(x+1)}$

Example: Adding and Subtracting Rational Expressions.

a) $\frac{5}{8} + \frac{7}{12} + \frac{11}{40}$ The LCD is 120. We first convert each fraction to an equivalent one with 120 as the denominator.

$$\frac{5}{8} = \frac{\qquad}{120}, \qquad \frac{7}{12} = \frac{\qquad}{120}, \qquad \frac{11}{40} = \frac{\qquad}{120}$$

Then, we add the resulting expressions:

$$\frac{\qquad}{120} + \frac{\qquad}{120} + \frac{\qquad}{120} = \frac{\qquad}{120}.$$

b) $\frac{3}{x} + \frac{1}{y}$

c) $\frac{3x + 2y}{3y} - \frac{x + 2y}{6x}$

Exercises: Adding and Subtracting Rational Expressions.

a) $\frac{a}{b} - \frac{c}{d}$

b) $\frac{y}{y+3} + \frac{9y+18}{y^2+3y}$

c) $\frac{4x-5}{x^2-7x+12} - \frac{x+7}{x^2+2x-15}$

d) $\frac{2x}{x^2-4} + \frac{5}{2-x} - \frac{1}{2+x}$

e) $\frac{2x-10}{x^2-25} - \frac{5-x}{25-x^2}$

f) $\frac{1-4x}{(4x-1)(2x+5)} - \frac{2-x}{(2x+5)(x-1)}$