

## Elementary Algebra Review for Test 1

### Fractions

**Multiplication of Fractions:** To multiply two fractions, we multiply the numerators and multiply the denominators:  $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ . Be sure to simplify.

**Exercise 1:** Multiply

a)  $\frac{2}{3} \cdot \frac{15}{14}$

b)  $\left(\frac{12}{13}\right)\left(\frac{4}{-3}\right)$

**Simplifying Fractions:** Factor out any common factors from both the numerator and denominator and then use the identity property of 1. The fraction is “reduced” if there are no more common factors.

**Exercise 2:** Simplify

a)  $\frac{18}{40}$

b)  $\frac{-28}{35}$

**Adding and Subtracting Fractions with Unlike Denominators:** When the denominators are not the same, we first have to find a common denominator, convert the fractions to equivalent fractions with the LCD as the new denominator and then add as before.

**Exercise 3:**

a) Add:  $\frac{2}{9} + \frac{3}{8}$

b) Subtract:  $\frac{7}{12} - \frac{3}{8}$

**Definition:** The **reciprocal** or **multiplicative inverse of a number**  $a$  is the number  $\frac{1}{a}$ .

**Dividing fractions:** Multiply the first fraction by the reciprocal of the second fraction. We usually say “invert and multiply.” Be sure to simplify if possible.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

**Exercise 4:** Division of Fractions

a) Divide:  $\frac{2}{9} \div \frac{3}{8}$

b) Divide:  $\frac{21}{16} \div \frac{3}{8}$

## Sets of Numbers

**Natural Numbers:** These are the counting numbers.  $N = \{1, 2, 3, 4, 5, \dots\}$

**Prime Numbers:** Numbers that have exactly two distinct factors, the number itself and 1.

$$P = \{2, 3, 5, 7, 11, 13, 17, 19, 23, \dots\}$$

**Whole numbers:** This set includes the natural numbers but it also includes 0.

$$W = \{0, 1, 2, 3, 4, 5, \dots\}$$

**Composite:** A natural number other than 1 that is not a prime number. Every composite number can be factored into a product of a prime numbers. This is called the **prime factorization** of that composite number.

Example: 6 is a composite number and its prime factorization is  $6 = 2 \cdot 3$ .

**Integers:** All whole numbers and their opposites:  $\mathbb{Z} = \{\dots - 3, -2, -1, 0, 1, 2, 3, \dots\}$

**Rational Numbers:** Numbers of the form  $\frac{p}{q}$  where  $p$  and  $q$  are integers. This set contains

fractions and terminating and repeating decimals in addition to the integers.

**Irrational numbers:** Numbers whose decimal expansions don't terminate or repeat.

**Real numbers:** Rational and irrational numbers together correspond to all the points on a number line.

**Exercise 5:** Consider the following set of numbers  $\left\{-3.5, -11, -9, 2\frac{2}{3}, \sqrt{2}, -\sqrt{2}, 2, -\frac{5}{6}\right\}$ . For each number in this set, identify which of the above sets of numbers to which it belongs.

## Rules for Operations on Real Numbers

**Definition: Absolute value** can be thought of as the distance a number is from 0 on a number line.  $|3| = 3$ ,  $|-3| = 3$ ,  $|0| = 0$

**Exercise 6:** Evaluate the following expressions.

a)  $|-11|$

b)  $-|-11|$

**Definition: Inequalities** are any expressions dealing with greater than or less than signs.

**Exercise 7:** Insert either  $<$ ,  $>$ , or  $=$  to make the statement true.

a)  $|-11|$   $|-10|$

b)  $-2.36$   $-2.35$

c)  $|-1.1|$   $1$

**Adding Positive and Negative numbers:** Subtract the absolute value of the smaller number from the absolute value of the larger number. The sign of the result is the same as the original sign of the number with the larger absolute value.

**Exercise 8:**

a)  $(-11) + (6)$

c)  $0 - (-5)$

b)  $-11 - (-6)$

d)  $-4 - 5$

**Opposites (or additive inverses):** Whenever opposites are added the result is 0 and whenever two numbers add to 0, those numbers are opposites.

### Exponents and the Order of Operations

**Exponential notation:** Exponents are shorthand for repeated multiplication.

Example:  $3 \cdot 3 \cdot 3 \cdot 3 = 3^4$  Here, 4 is the **exponent** and 3 is the **base**.

**Exercise 9:** Evaluate each expression.

a)  $-5^2$

b)  $(-5)^2$

c)  $-(-5)^2$

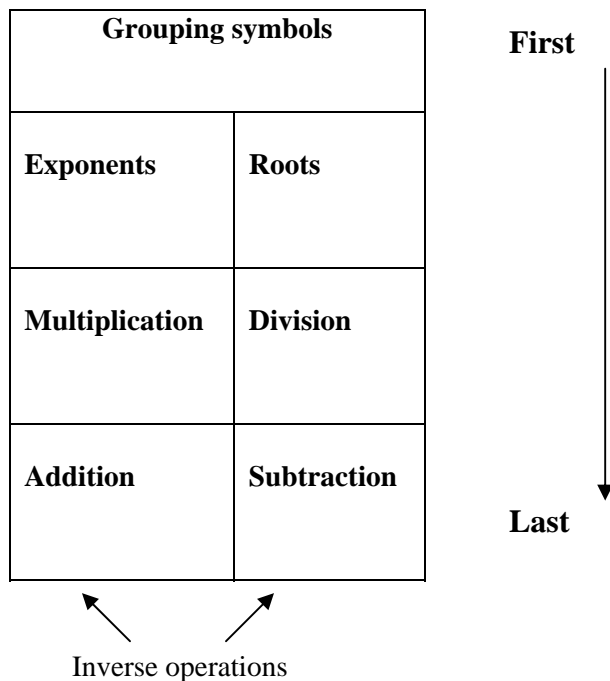
d)  $-(-5^2)$

e)  $\left(\frac{2}{3}\right)^3$

f)  $\left(\frac{-2}{3}\right)^2$

### Order of Operations:

1. Perform all calculations within grouping symbols
2. Simplify all exponential expressions (and roots)
3. Perform all multiplication and division, in order, from left to right.
4. Perform all addition and subtraction, in order from left to right.



**Exercise 10:** Evaluate each expression.

a)  $5 + 3 \cdot 4$

b)  $(5 - 2)^2 + 3 \cdot 5$

c)  $5 - 3^2 \cdot 4$

d)  $[(5 - 2)^2 + 3] \cdot 5 - 4$

e)  $4^3 \div 4^2 - 5(2 - 7) \div 5$

## Evaluating Variable Expressions

**Variable:** A letter that can represent different numbers.

**Constant:** A number in an equation that stays the same.

**Algebraic Expression:** Consists of variables and/or numerals, often with operation signs and grouping symbols:  $3a(b + c)$

**Value:** The number you get when evaluating an expression.

**Exercise 11:** Evaluate each expression for the given value of the variable.

a)  $3x - 5 + 4x$  for  $x = -2$

b)  $3x^2 - 6x - 4$  for  $x = -1$

c)  $3x^2 - 6x - 4$  for  $x = -2$

d)  $(3x^2 - 1)(x - 1) + 5$  for  $x = -2$

e)  $3(x + y)^3 + 2(x + y)^2 - 4(x + y)$  for  $x = -3$  and  $y = 1$

## Division and Zero

**Fact:** 0 divided by any non-zero number  $a$  is 0:  $\frac{0}{a} = 0$

**Division by Zero:** Suppose  $a$  is any non-zero number. Then:

a)  $\frac{a}{0}$  is undefined.

b)  $\frac{0}{0}$  is indeterminate.

## The Properties of Real Numbers

**Commutative Property of Addition (CPA):** For any real numbers  $a$  and  $b$ ,  $a + b = b + a$ .

**Commutative Property of Multiplication (CPM):** For any real numbers  $a$  and  $b$ ,  $a \cdot b = b \cdot a$ .

**Associative Property of Addition (APA):** For any real numbers  $a$ ,  $b$ , and  $c$ :

$$(a + b) + c = a + (b + c).$$

**Associative Property of Multiplication (APM):** For any real numbers  $a$ ,  $b$ , and  $c$ :

$$(a \cdot b) \cdot c = a \cdot (b \cdot c).$$

**The Distributive Property:** For any real numbers  $a$ ,  $b$ , and  $c$ :

$$a(b + c) = a \cdot b + a \cdot c$$



If we perform the multiplication  $a(b + c) = a \cdot b + a \cdot c$ , then we are **distributing**. If we start with the sum and get a product  $a \cdot b + a \cdot c = a(b + c)$ , then we are **factoring out** the common factor  $a$ .

**Exercise 12:** Complete the statement using the given property:

- a) Commutative Property of Addition  $x + 3 =$
- b) Commutative Property of Multiplication  $5(x + 3) =$
- c) Associative Property of Addition  $(x + 3) + 5 =$
- d) Associative Property of Multiplication  $(3 \cdot x)y =$
- e) Distributive Property:  $5(x + 3) =$

**Exercise 13:** Name the property.

- a)  $x + (y + 3) = (y + 3) + x$
- b)  $(3 + x)5 = 5(3 + x)$
- c)  $(3 + x)5 = 3 \cdot 5 + x \cdot 5$
- d)  $(x + 5) + 4 = x + (5 + 4)$

### Algebraic Expressions - Combining Like Terms

**Like terms** are either constant terms or terms containing the same variable raised to the same powers.

#### Combining Like Terms

We use the distributive property in order to combine like terms. We factor out the common factor and add the numerical coefficients.

**Example:**  $5x + 3x = (5 + 3)x = 8x$

**Exercise 14:**

- a)  $4x + 3x - 5x - 10$
- b)  $2x - 6 - 4x - 11$
- c)  $\frac{1}{3}x + 3 - \frac{1}{2}x - 5$
- d)  $4.2 - 2.7x + 2.2 + 3.1x + 8.9$
- e)  $\frac{2}{3}x - 2 - \frac{3}{4}x - 1$

#### Using the distributive property to remove parentheses

**Example:**  $-(5x + 3) = -5x - 3$

**Exercise 15: Distribute and Combine Like Terms**

- a)  $-3(5 - 3x)$
- b)  $-2(3x - 2 + 5y)$
- c)  $-1.9(-2 + 3x - 2.1y)$

**Exercise 16: Simplify.**

- a)  $-3(2 - x) - 4(-2x + 1)$
- b)  $-(2 - x) - 3(-3x + 2)$
- c)  $-\frac{1}{3}(3 - 6x) - \frac{1}{2}(-2x - 4)$

**Solving Linear Equations**

**Definition:** An **equation** is a statement about the relationship between two algebraic expressions.

The truth of an equation may depend upon the value of the variable or variables. Such statements are called **conditional statements**. Some equations are true (or false) regardless of the value assigned to the variable. These are called **identities**.

**Definitions:** A **linear equation** in one variable is an equation that can be written in the form  
$$ax + b = 0.$$

Here  $a$  and  $b$  are real numbers with  $a \neq 0$ .

**Solving** a linear equation means finding the value(s) of the variable that make the equation true. The **solution** is the set of values of the variable that make the equation true.

**Exercise 17:** Is  $x = \frac{1}{2}$  a solution of the equation  $x + 3 = 2 + 3x$ ?

**The Addition Property of Equality:** If  $a = b$  then  $a + c = b + c$ . This says that you can add the same number to both sides of an equation without changing the solution.

**Example:** If  $x - 5 = 7$  then we can add 5 to both sides of the equation:

$$\begin{aligned}x - 5 &= 7 \\x - 5 + 5 &= 7 + 5 && \text{Addition property} \\x + 0 &= 12 \\x &= 12\end{aligned}$$

**Exercise 18:** Solving Equations Using the Addition Property of Equality.

- a)  $7.25 = 9.75 + x$
- b)  $x + 8.1 = 8.01$
- c)  $-6 + x = -7$
- d)  $x - 8.77 = -18$

**The Multiplication Property of Equality:** If  $a = b$  then  $ac = bc$ . This says that you can multiply both sides of an equation by the same number without changing the solution.

**Exercise 19:** Solving Equations Using the Multiplication Property of Equality.

a)  $\frac{x}{3} = 12$

b)  $\frac{2}{5}x = -6$

c)  $2x = -14$

d)  $0.75x = -1.5$

e)  $-2x = \frac{2}{3}$

**Definition:** Two equations are called **equivalent equations** if they have the same solution.

**Exercise 20:** Solving Equations using both Properties of Equality.

a)  $2x - 7 = x + 4$

b)  $5x + 6 = 2x + 3 - x$

c)  $0.25x + 1.2 = 1.55 - 1.2x$

d)  $7x + 3 = 3x - 17$

e)  $-\frac{2}{3}x + 3 = \frac{3}{4}$

## Solving Equations Containing Grouping Symbols

To solve a linear equation in one variable:

1. Simplify to terms, that is, remove grouping symbols.
2. Combine like terms on each side of the equation  
Note: You may need to repeat the first two steps several times to remove *all* the grouping symbols.
3. Move any terms that contain the variable to one side by adding inverses to both sides of the equation. (Addition Property of Equality)
4. Isolate the variable:
  - a. First, move all constants being added to or subtracted from the variable term to the other side. (Addition Property of Equality)
  - b. Finally, complete the solution by multiplying both sides of the equation by the reciprocal of the coefficient of the variable. (Multiplication Property of Equality)
5. Check the solution – Replace the variable with the value you found and simplify both sides of the equation. If the resulting number on each side of the equation is the same, then the solution is correct.

**Exercise 21: Solving Equations with Grouping Symbols**

- a)  $12(x + 4) = 3(-6x + 10)$                       e)  $2x - 5(x - 2) = 20 - (2x + 6)$   
b)  $0 = 3(x + 2) + 3(x - 5)$   
c)  $5(x + 1) - 3(2x + 3) = 3(x + 2)$   
d)  $3(x - 1) + 9 = 8x + 6 - 5x$

**Exercise 22: Formulas**

- a) Solve for  $m$ :  $y = mx + b$                       d) Solve for  $P$ :  $A = P + Prt$   
b) Solve for  $d$ .  $F = \frac{c + 2d}{3}$                       e) Solve for  $l$ :  $P = 2l + 2w$   
c) Solve for  $t$ :  $A = P + Prt$   
f) Given  $C = \frac{5}{9}(F - 32)$ , find  $F$  when  $C = 71$   
g) Given  $A = P(1 + rt)$ , find  $A$  when  $t = 5$ ,  $r = 0.05$ , and  $P = 1000$ .  
h) Given  $y = mx + b$  find  $x$  when  $y = 15$ ,  $b = 5$ , and  $m = 2$ .

**Translation****Exercise 23: Translate to a mathematical expression carefully labeling the variable.**

- a) Three less than twice a number  
b) The difference between five and a number.  
c) Five times the sum of seven and a number.  
d) One more than four times a number.

**Exercise 24: Word Problems**

1. If three more than three-fifths of a number is one less than that number, what is the number?
2. Your bill for lunch was \$13.23. This included 8% sales tax. What was the cost of the meal without tax?
3. After a 25% discount, a new CD player was selling for \$146.25. What was the original price of the CD player?
4. The second angle of a triangle is twice as large as the first. The third angle is  $30^\circ$  more than the sum of the first two angles. What is the measure of each angle?
5. Why are you required to take Algebra? What is your motivation to succeed?
6. Suppose the width of a rectangle is  $\frac{1}{2}$  the length and the perimeter is 36 feet. What are the dimensions of the rectangle?
7. Consumer experts advise us to never pay the sticker price for a car. A rule of thumb is to pay the sticker price minus 20% of the sticker price, plus \$200. A car is purchased for \$11,520 using the rule. What was the sticker price?
8. Two angles are complementary. If the measure of one angle is six degrees less than twice the measure of the second, find the measure of each angle.
9. Two angles are supplementary. If the measure of one angle is five degrees less than four times the measure of the second, find the measure of each angle.

10. The measure of the second angle of a triangle is twice the measure of the first. The measure of the third angle of a triangle is eight less than the measure of the first. Find the measure of all three angles.

**Elementary Algebra Review for Test 1 Answer Key**

**Exercise 1: Multiply**

a)  $\frac{2}{3} \cdot \frac{15}{14} = \frac{5}{7}$

b)  $\left(\frac{12}{13}\right)\left(\frac{4}{-3}\right) = -\frac{16}{13}$

**Exercise 2: Simplify**

a)  $\frac{18}{40} = \frac{9}{20}$

b)  $\frac{-28}{35} = -\frac{4}{5}$

**Exercise 3: Perform the indicated operation.**

a)  $\frac{2}{9} + \frac{3}{8} = \frac{43}{72}$

b)  $\frac{7}{12} - \frac{3}{8} = \frac{5}{24}$

**Exercise 4: Division of Fractions**

a)  $\frac{2}{9} \div \frac{3}{8} = \frac{16}{27}$

b)  $\frac{21}{16} \div \frac{3}{8} = \frac{7}{2}$

**Exercise 5: Natural Numbers: 2; Prime Numbers: 2; Whole numbers: 2 ; Composite: none**

**Integers: -11,-9, 2; Rational Numbers: -3.5, -11, -9,  $2\frac{2}{3}$ , 2,  $-\frac{5}{6}$**

**Irrational numbers:  $\sqrt{2}$ ,  $-\sqrt{2}$  ; Real numbers: They are all real numbers.**

**Exercise 6: Evaluate the following expressions.**

a)  $|-11| = 11$

b)  $-|-11| = -11$

**Exercise 7: Insert either  $<$ ,  $>$ , or  $=$  to make the statement true.**

a)  $|-11| > |-10|$

c)  $|-1.1| > 1$

b)  $-2.36 < -2.35$

**Exercise 8:**

a)  $(-11) + (6) = -5$

b)  $-11 - (-6) = -5$

c)  $0 - (-5) = 5$

d)  $-4 - 5 = -9$

**Exercise 9: Evaluate each expression.**

a)  $-5^2 = -25$

b)  $(-5)^2 = 25$

c)  $-(-5)^2 = -25$

d)  $-(-5^2) = 25$

e)  $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

f)  $\left(\frac{-2}{3}\right)^2 = \frac{4}{9}$

**Exercise 10:** Evaluate each expression.

a)  $5 + 3 \cdot 4 = 17$

b)  $(5 - 2)^2 + 3 \cdot 5 = 24$

c)  $5 - 3^2 \cdot 4 = -31$

d)  $[(5 - 2)^2 + 3] \cdot 5 - 4 = 56$

e)  $4^3 \div 4^2 - 5(2 - 7) \div 5 = 9$

**Exercise 11:** Evaluate each expression for the given value of the variable.

a)  $3x - 5 + 4x = -19$  when  $x = -2$

b)  $3x^2 - 6x - 4 = 5$  when  $x = -1$

c)  $3x^2 - 6x - 4 = 20$  when  $x = -2$

d)  $(3x^2 - 1)(x - 1) + 5 = -28$  when  $x = -2$

e)  $3(x + y)^3 + 2(x + y)^2 - 4(x + y) = -8$  when  $x = -3$  and  $y = 1$

**Exercise 12:** Complete the statement using the given property:a) Commutative Property of Addition  $x + 3 = 3 + x$ b) Commutative Property of Multiplication  $5(x + 3) = (x + 3)5$ c) Associative Property of Addition  $(x + 3) + 5 = x + (3 + 5)$ d) Associative Property of Multiplication  $(3 \cdot x)y = 3(xy)$ e) Distributive Property:  $5(x + 3) = 5x + 5 \cdot 3$ **Exercise 13:** Name the property.

a)  $x + (y + 3) = (y + 3) + x$  Commutative Property of Addition

b)  $(3 + x)5 = 5(3 + x)$  Commutative Property of Multiplication

c)  $(3 + x)5 = 3 \cdot 5 + x \cdot 5$  Distributive

d)  $(x + 5) + 4 = x + (5 + 4)$  Associative Property of Addition

**Exercise 14:**

a)  $4x + 3x - 5x - 10 = 2x - 10$

b)  $2x - 6 - 4x - 11 = -2x - 17$

c)  $\frac{1}{3}x + 3 - \frac{1}{2}x - 5 = -\frac{1}{6}x - 2$

d)  $4.2 - 2.7x + 2.2 + 3.1x + 8.9 = 0.4x + 15.3$

e)  $\frac{2}{3}x - 2 - \frac{3}{4}x - 1 = -\frac{1}{12}x - 3$

**Exercise 15: Distribute and Combine Like Terms**

a)  $-3(5 - 3x) = -15 + 9x$



g)  $A = 1250$

h)  $y = 5$

**Translation****Exercise 23: Translate to a mathematical expression carefully labeling the variable.****Let  $x =$  the number, then**

- a) Three less than twice a number is  $2x - 3$
- b) The difference between five and a number is  $5 - x$
- c) Five times the sum of seven and a number is  $5(x + 7)$
- d) One more than four times a number =  $4x + 1$

**Exercise 24: Word Problems**

1. If three more than three-fifths of a number is one less than that number, what is the number?

Answer:  $\frac{3}{5}x + 3 = x - 1$  ; The number is 10.

2. Your bill for lunch was \$13.23. This included 8% sales tax. What was the cost of the meal without tax?

Answer:  $x + 0.08x = 13.23$  ; Lunch cost \$12.25.

3. After a 25% discount, a new CD player was selling for \$146.25. What was the original price of the CD player?

Answer:  $x - 0.25x = 146.25$  ; The player cost \$195.

4. The second angle of a triangle is twice as large as the first. The third angle is  $30^\circ$  more than the sum of the first two angles. What is the measure of each angle?

Answer:  $x + 2x + (3x + 30) = 180$  ; The angles are  $25^\circ$  ,  $50^\circ$  , and  $105^\circ$  .

5. Why are you required to take Algebra? What is your motivation to succeed?

6. Suppose the width of a rectangle is  $\frac{1}{2}$  the length and the perimeter is 36 feet. What are the dimensions of the rectangle?

Answer:  $2l + 2\left(\frac{1}{2}l\right) = 36$  , The length is 12 ft and the length is 6 ft.

7. Consumer experts advise us to never pay the sticker price for a car. A rule of thumb is to pay the sticker price minus 20% of the sticker price, plus \$200. A car is purchased for \$11,520 using the rule. What was the sticker price?

Answer:  $x - 0.2x + 200 = 11520$  , The sticker price was \$14,150.

8. Two angles are complementary. If the measure of one angle is six degrees less than twice the measure of the second, find the measure of each angle.

Answer:  $x + 2x - 6 = 90$  , The angles are  $32^\circ$  , and  $58^\circ$  .

9. Two angles are supplementary. If the measure of one angle is five degrees less than four times the measure of the second, find the measure of each angle.

Answer:  $x + 4x - 5 = 180$  , The angles are  $37^\circ$  , and  $143^\circ$  .

10. The measure of the second angle of a triangle is twice the measure of the first. The measure of the third angle of a triangle is eight less than the measure of the first. Find the measure of all three angles.

Answer:  $x + 2x + (x - 8) = 180$  ; The angles are  $47^\circ$  ,  $94^\circ$  , and  $39^\circ$  .